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Consistent Determination of Antecedent Soil Moisture (Retention Parameter) of the SCS Method Based on Stochastic Soil Water Balance

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Conselho Nacional de Desenvolvimento Científico e Tecnológico

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Motivation and Overview

- The **SCS curve number** is a widely used (including in the SWAT model) empirical method for runoff quantification
- Based on extensive observations, it provides robust estimates of runoff at the event scale
- However, the antecedent soil moisture conditions (retention parameter) are linked to soil, plant and climatic conditions through empirical tables and charts having little physical basis
- The **stochastic soil moisture models used in ecohydrology** (e.g., Porporato et al. AWR, 2002; Rodriguez-Iturbe and Porporato, Ecohydrology, Cambridge University Press, 2004) provide probabilistically consistent determination of antecedent soil moisture conditions based on soil, plant and climate characteristics but with rather crude runoff representation

→ **Combining the theoretical framework of stochastic ecohydrology methods with the empirical strength of the SCS method**

SCS Curve Number Method: Background

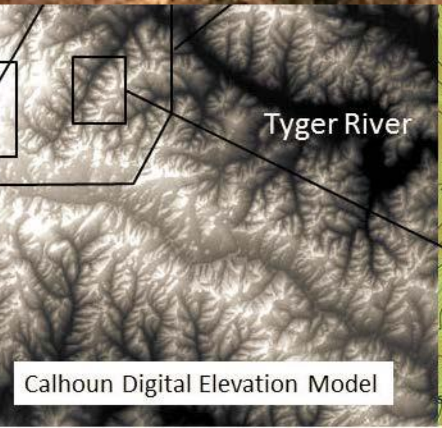
After the Dust Bowl of the ‘30s, the US Congress declares soil erosion “a national menace”, establishing the Soil Conservation Service (USDA) to develop extensive conservation programs that retain topsoil and prevent irreparable damage to the land.

Dust storm approaching Stratford, Texas, April 18 1935

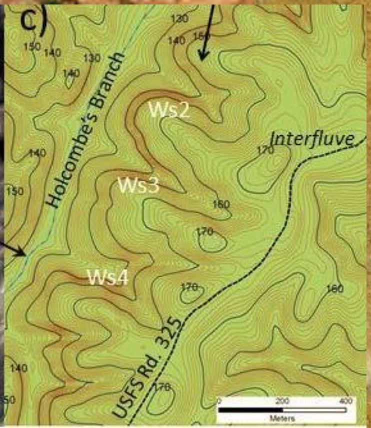
Buried machinery in barn lot, Dallas, South Dakota, May 1936

The issue is still very actual, multidisciplinary and complex, involving ecohydrology, geomorphology, plant physiology, biogeochemistry, etc.

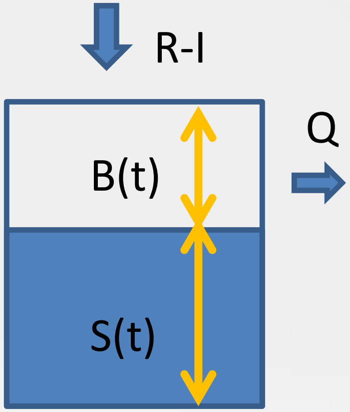
Calhoun, 1950 circa



Calhoun Digital Elevation Model



SCS Curve Number



- The SCS curve number equation for runoff is

$$Q = \frac{R - I}{R - I + B}$$

where:

Q = depth of runoff

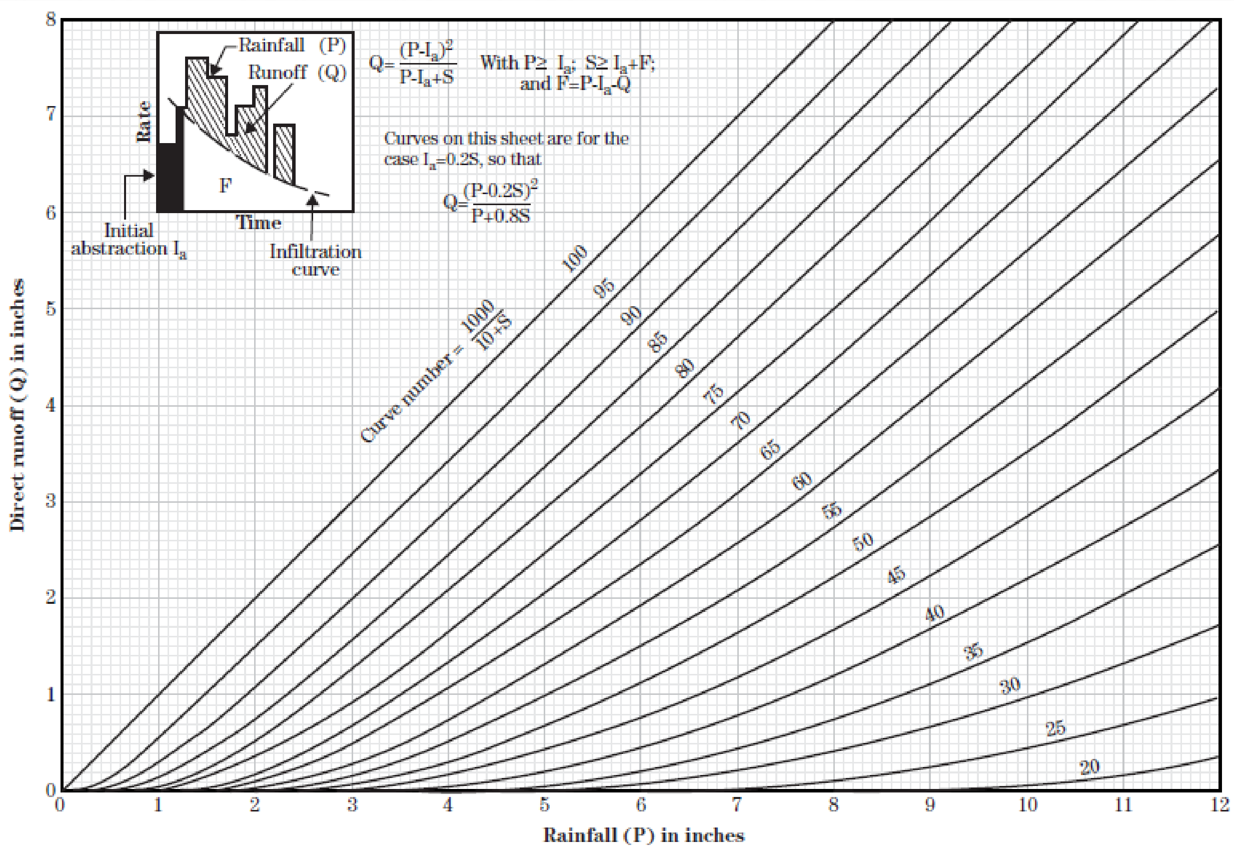
R = depth of rainfall

I = initial abstraction

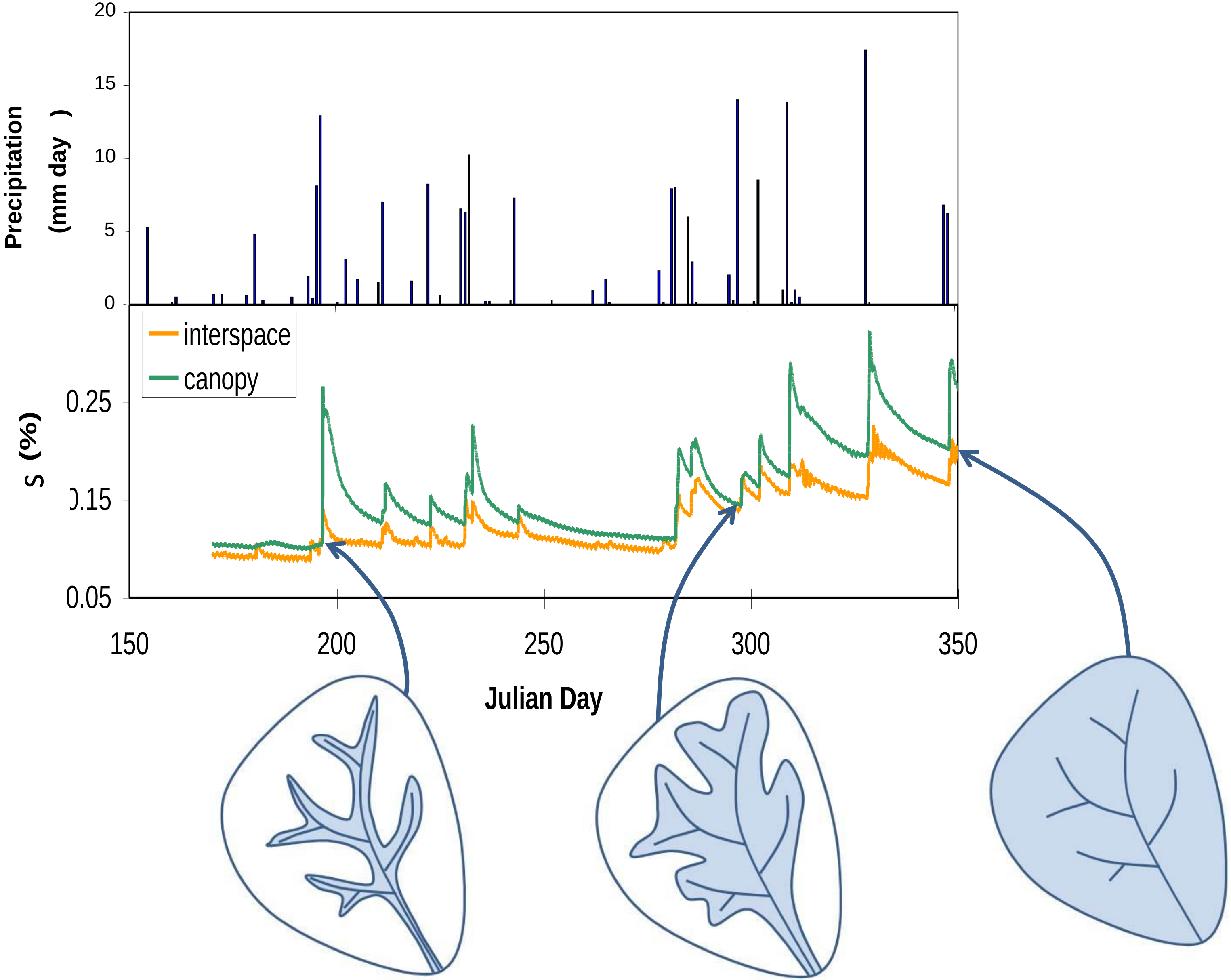
B = basin ret. par.

$$CN = \frac{1000}{10 + \frac{B\ (\text{mm})}{25.4}}$$

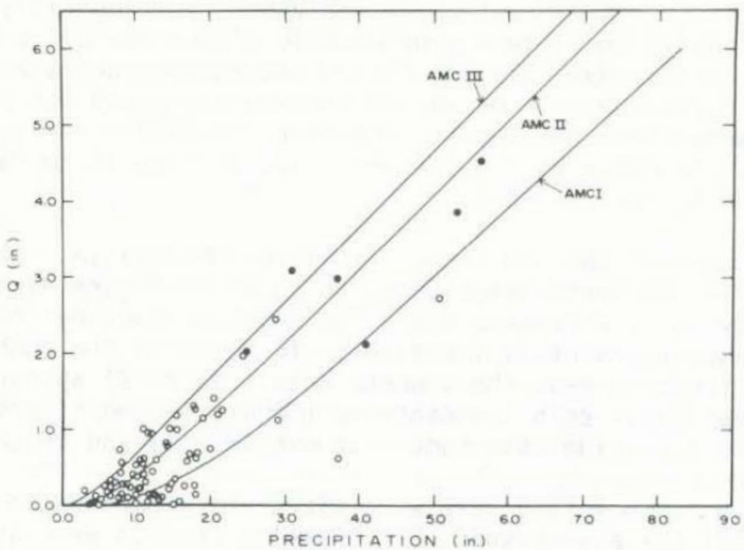
Part 630 Hydrology, National Engineering Handbook



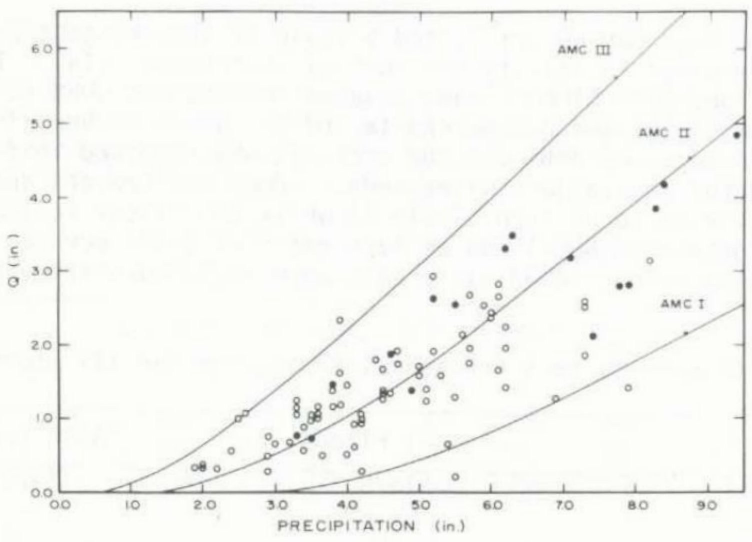
Lumped parameterization of spatial response of watershed due to different soil moisture conditions



Curve number data



Observed rainfall-runoff events for Falling Rock Watershed, Kentucky, annual flood-derived AMC II CN and AMC I-III envelope CNs from Table 10.1 of NEH-4 (Solid dots are annual rainstorm floods).



Observed rainfall-runoff events for Coweeta Watershed No. 36, North Carolina, annual flood-derived AMC II CN and AMC I-III envelope CNs from Table 10.1 of NEH-4 (Solid dots are annual rainstorm floods).

Curve number selection

Figure 9–1 Estimating runoff curve numbers of forest-range complexes in Western United States: herbaceous and oak-aspen complexes

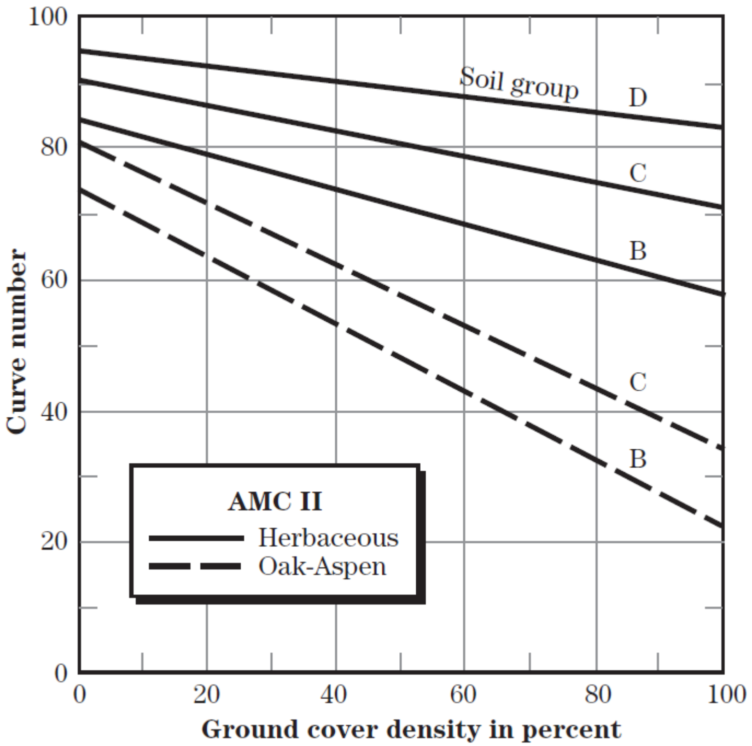
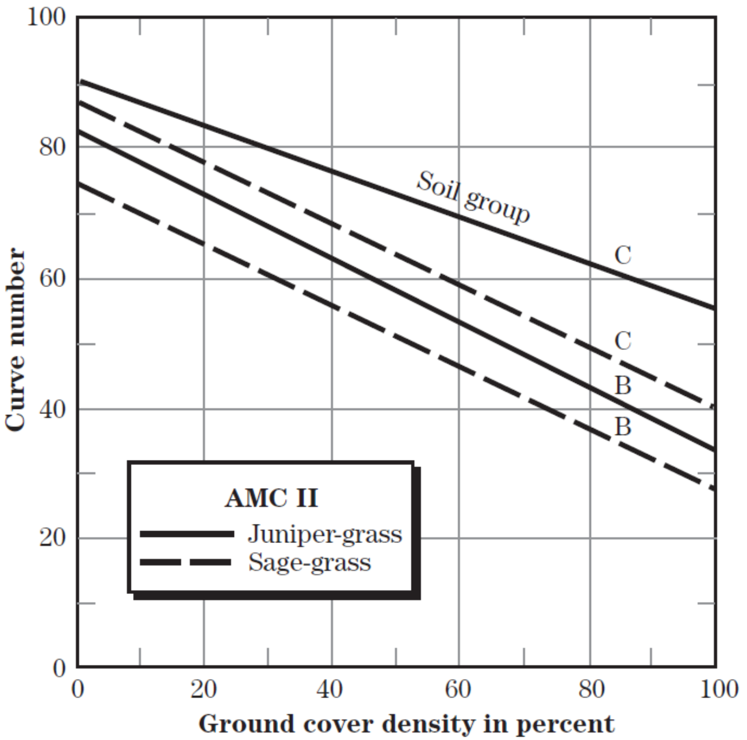


Figure 9–2 Estimating runoff curve numbers of forest-range complexes in Western United States: juniper-grass and sage-grass complexes



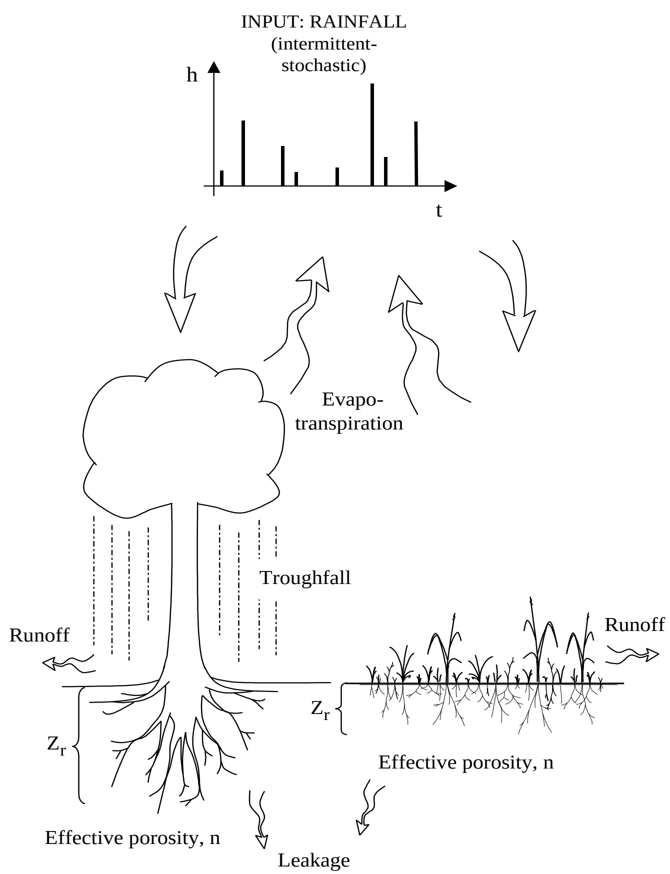
Tables for antecedent moisture conditions

Table 10–1 Curve numbers (CN) and constants for the case $I_a = 0.2S$

1	2	3	4	5	1	2	3	4	5
CN for ARC II	- - CN for ARC - - I	III	S values* (in)	Curve* starts where P = (in)	CN for ARC II	- - CN for ARC - - I	III	S values* (in)	Curve* starts where P = (in)
100	100	100	0	0	60	40	78	6.67	1.33
99	97	100	.101	.02	59	39	77	6.95	1.39
98	94	99	.204	.04	58	38	76	7.24	1.45
97	91	99	.309	.06	57	37	75	7.54	1.51
96	89	99	.417	.08	56	36	75	7.86	1.57
95	87	98	.526	.11	55	35	74	8.18	1.64
94	85	98	.638	.13	54	34	73	8.52	1.70
93	83	98	.753	.15	53	33	72	8.87	1.77
92	81	97	.870	.17	52	32	71	9.23	1.85
91	80	97	.989	.20	51	31	70	9.61	1.92
90	78	96	1.11	.22	50	31	70	10.0	2.00
89	76	96	1.24	.25	49	30	69	10.4	2.08
88	75	95	1.36	.27	48	29	68	10.8	2.16
87	73	95	1.49	.30	47	28	67	11.3	2.26
86	72	94	1.63	.33	46	27	66	11.7	2.34
85	70	94	1.76	.35	45	26	65	12.2	2.44
84	68	93	1.90	.38	44	25	64	12.7	2.54
83	67	93	2.05	.41	43	25	63	13.2	2.64
82	66	92	2.20	.44	42	24	62	13.8	2.76
81	64	92	2.34	.47	41	23	61	14.4	2.88
80	63	91	2.50	.50	40	22	60	15.0	3.00
79	62	91	2.66	.53	39	21	59	15.6	3.12
78	60	90	2.82	.56	38	21	58	16.3	3.26
77	59	89	2.99	.60	37	20	57	17.0	3.40
76	58	89	3.16	.63	36	19	56	17.8	3.56
75	57	88	3.33	.67	35	18	55	18.6	3.72
74	55	88	3.51	.70	34	18	54	19.4	3.88
73	54	87	3.70	.74	33	17	53	20.3	4.06
72	53	86	3.89	.78	32	16	52	21.2	4.24
71	52	86	4.08	.82	31	16	51	22.2	4.44
70	51	85	4.28	.86	30	15	50	23.3	4.66
69	50	84	4.49	.90	25	12	43	30.0	6.00
68	48	84	4.70	.94	20	9	37	40.0	8.00
67	47	83	4.92	.98	15	6	30	56.7	11.34
66	46	82	5.15	1.03	10	4	22	90.0	18.00
65	45	82	5.38	1.08	5	2	13	190.0	38.00
64	44	81	5.62	1.12	0	0	0	infinity	infinity
63	43	80	5.87	1.17					
62	42	79	6.13	1.23					
61	41	78	6.39	1.28					

* For CN in column 1.

Stochastic soil water balance used in ecohydrology



$$nZ_r \frac{ds(t)}{dt} = R(t) - I(t) - Q[s(t),t] - E[s(t)] - L[s(t)]$$

n = porosity

Z_r = active soil depth

$s(t)$ = relative soil moisture

$R(t)$ = rainfall rate

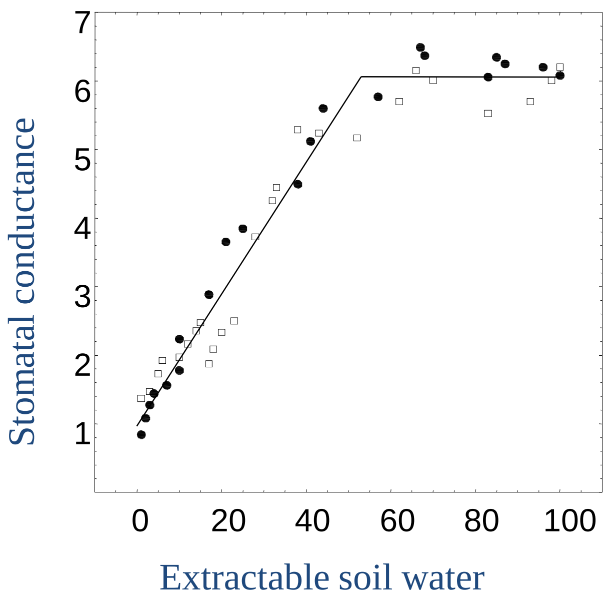
$I(t)$ = canopy interception

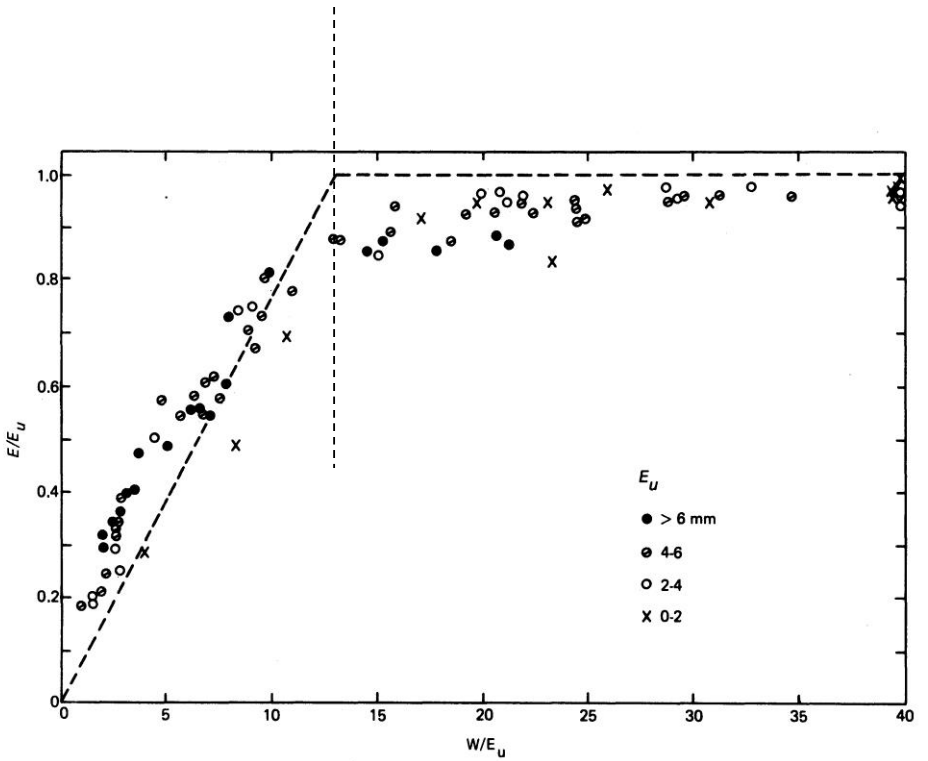
$Q[s(t),t]$ = runoff

$E[s(t),t]$ = evapotranspiration

$L[s(t),t]$ = leakage

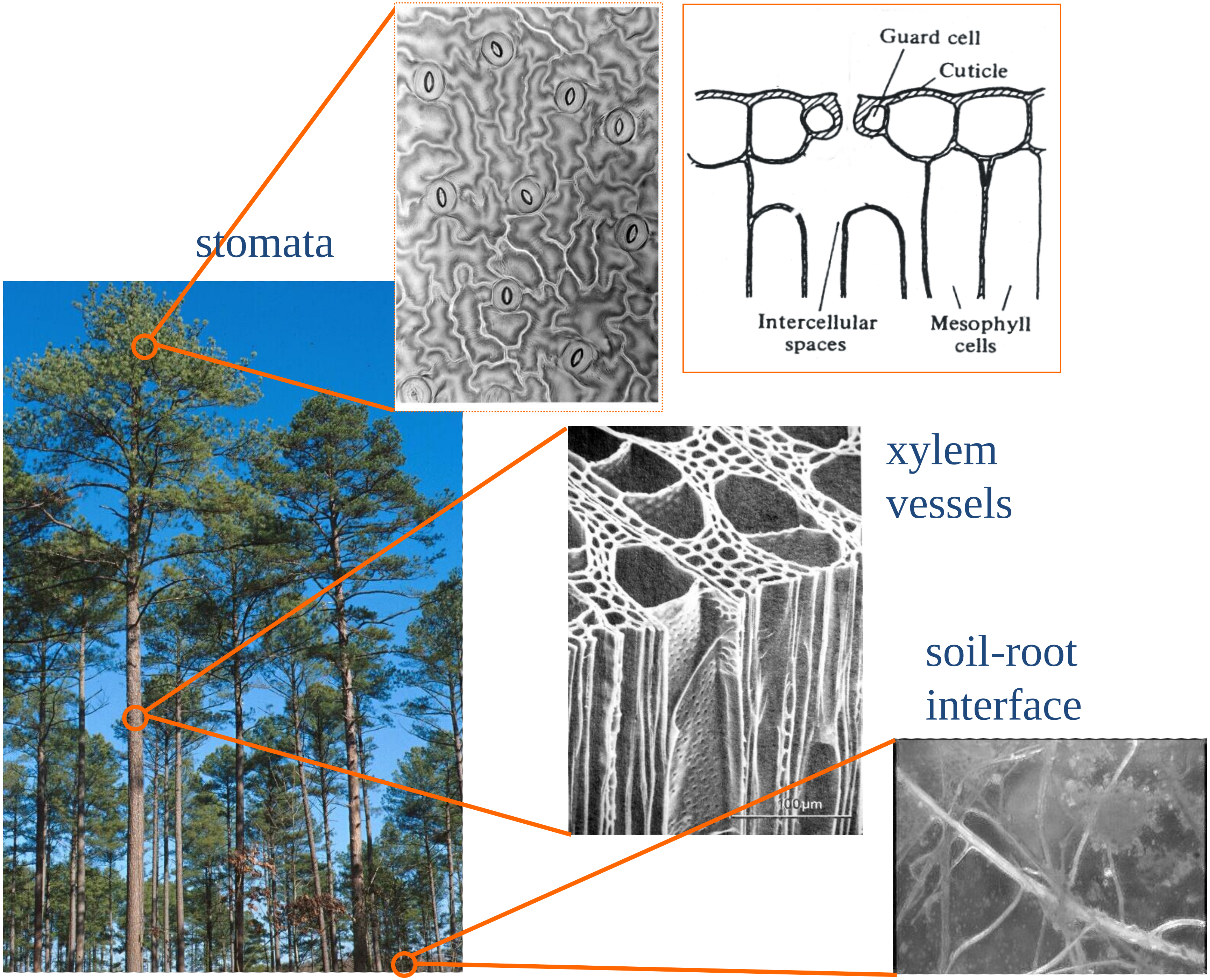
Transpiration





Adapted from: Gollan et al., *Oecologia*, 65, 356-362, 1985.

After: Federer, *Water Resour. Res.*, 15 (3), 555-561, 1979.

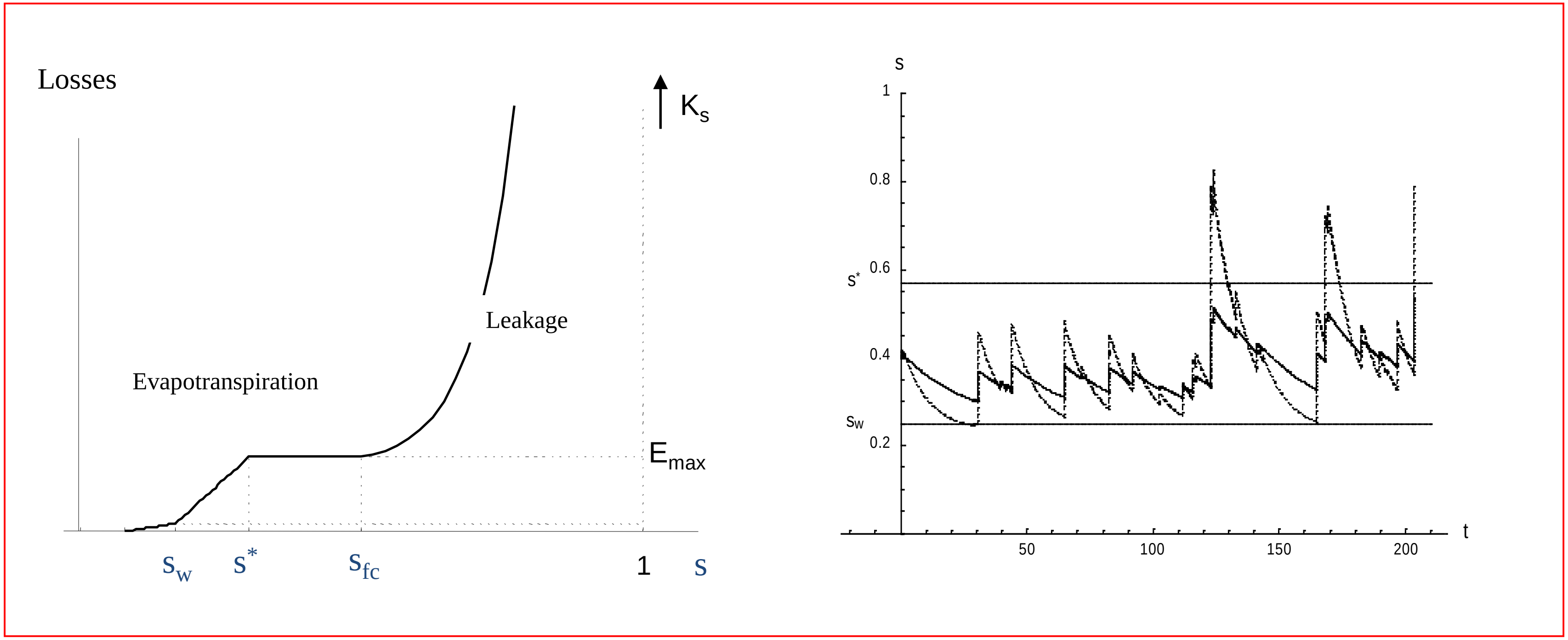

stomata
Guard cell
Cuticle
Intercellular spaces
Mesophyll cells
xylem vessels
100µm
soil-root interface

1-D Non-linear (stochastic) differential equation driven by a state-dependent Poisson noise

$$nZ_r \frac{ds(t)}{dt} = R(t) - I(t) - Q[s(t),t] - E[s(t)] - L[s(t)]$$

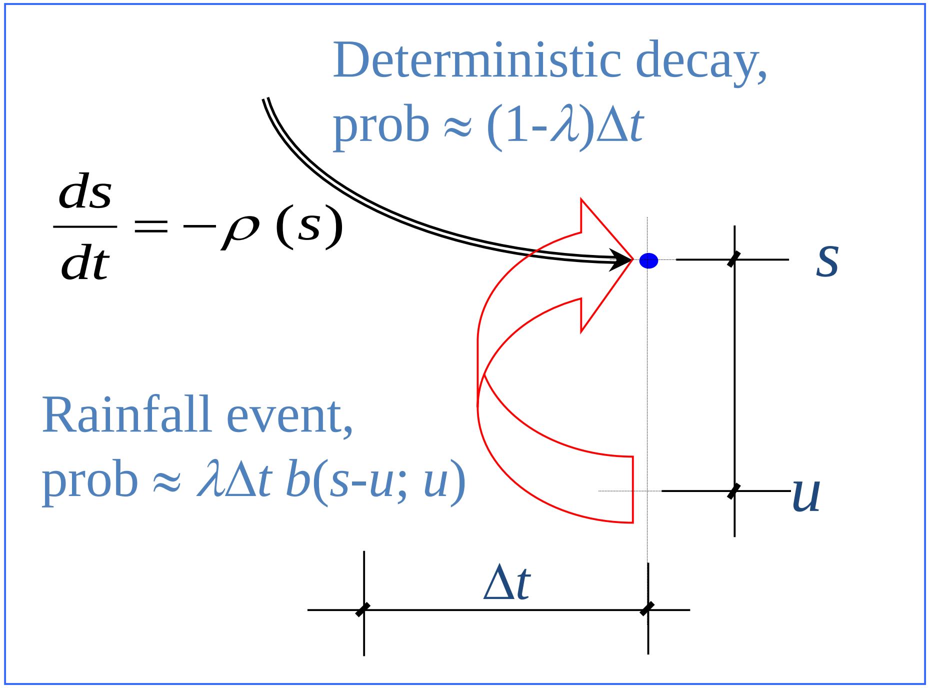
Rnd jump input

Nonlinear losses



Probabilistic description: Chapman-Kolmogorov Eq.

$$p(s,t+\Delta t) = \underbrace{(1-\lambda\,\Delta t)\,p(s+\Delta s,t)\,\mathrm{d}(s+\Delta s)}_{\text{det.decay}} + \underbrace{\lambda\,\Delta t \int_0^s p(u+\Delta u,t)\,p_y(s-u;u)\,\mathrm{d}(u+\Delta u)\,\mathrm{d}s}_{\text{jumps}}$$



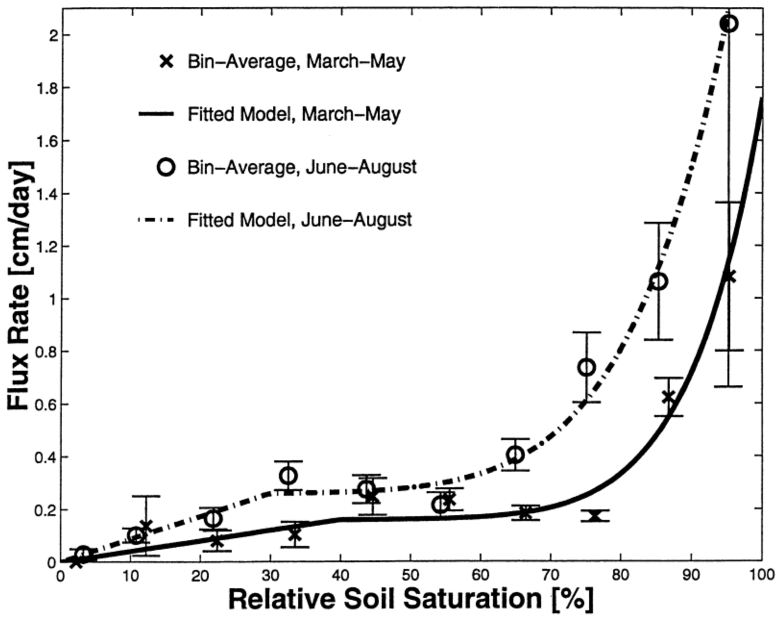
$$\frac{\partial}{\partial t}p(s,t) = \frac{\partial}{\partial s}\left[p(s,t)\rho(s)\right] - \lambda\,p(s,t) + \lambda\int_0^s p(u,t)\,\gamma\; e^{-\gamma(s-u)}du$$

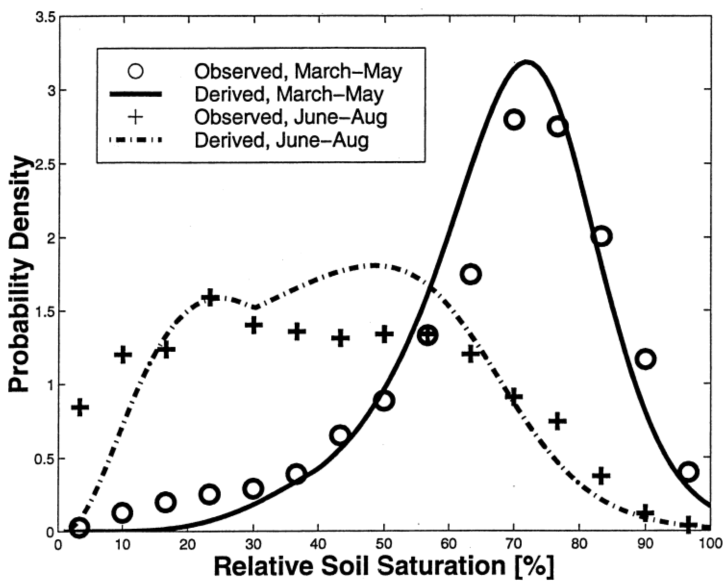
Steady-state PDF of soil moisture

$$p(s) = \frac{c}{\rho(s)} Exp\left(-\gamma s + \lambda \int_s \frac{du}{\rho(u)}\right)$$

Rodriguez-Iturbe et al., Proc. Royal Soc. A, 455, 3789, 1999
Porporato et al. Am. Nat., 2004
Daly and Porporato PRE 2010

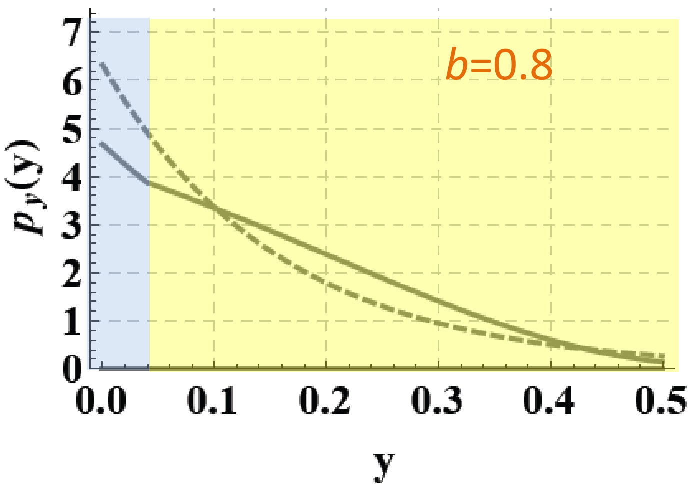
Experimental verification





Salvucci G. (2001) Water Resour. Res. 37(5), 1357-1365.

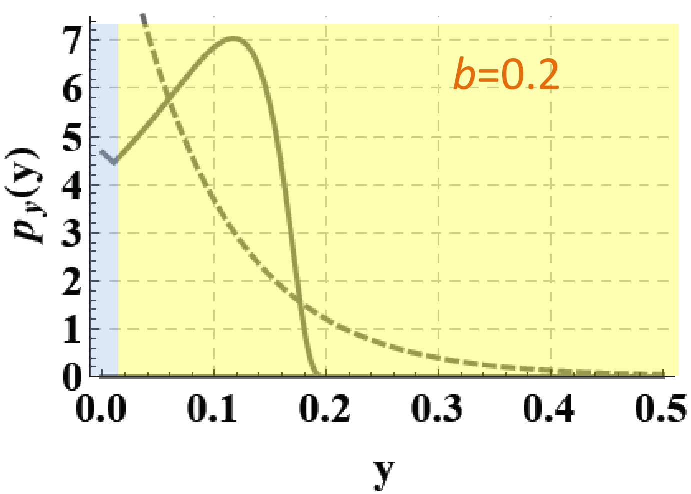
Jump (infiltration) distribution for SCS method



$$p_y(y|b) = \begin{cases} p_z(y) & y \leq \beta b \\ \frac{b^2}{(b-(y-\beta b))^2} p_z\left(\beta b + \frac{b(y-\beta b)}{b-(y-\beta b)}\right) & y > \beta b \end{cases}$$

For an exponential distribution of rainfall, i.e.,

$$p_z(z) = \gamma e^{-\gamma z}$$



The average infiltration is

$$\langle y|b\rangle = \frac{e^{\beta\gamma b} - 1 + \gamma b + b^2 e^{\gamma b}\gamma^2(\mathrm{Chi}(\gamma b) + \mathrm{Shi}(\gamma b))}{\gamma e^{\beta\gamma b}}$$

Approximate infiltration with a new exponential distribution, i.e.,

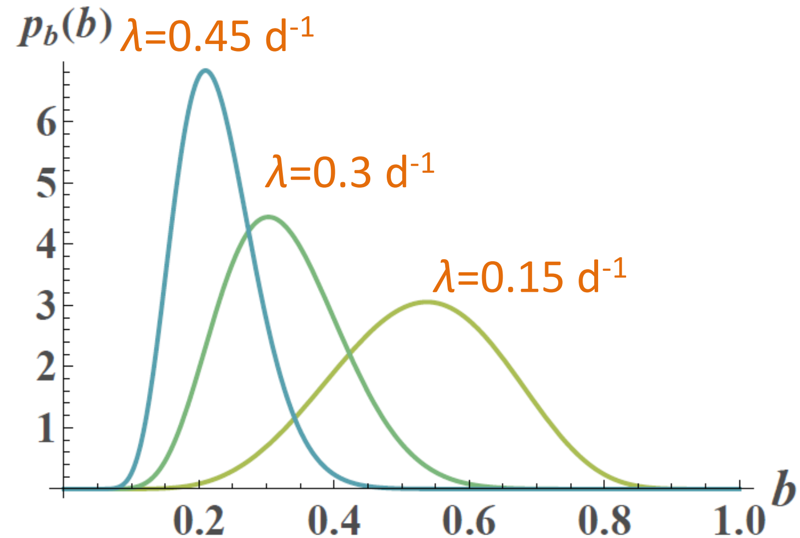
$$p_y(y|b) = \frac{1}{\langle y|b\rangle} e^{-\frac{1}{\langle y|b\rangle} y}$$

PDF of retention parameter

$$p_b(b) = C\,\frac{\langle y|b\rangle^{-\lambda/k}}{\Gamma(\lambda/k)}\,e^{-(1-b)/\langle y|b\rangle}(1-b)^{\frac{\lambda}{k}-1}$$

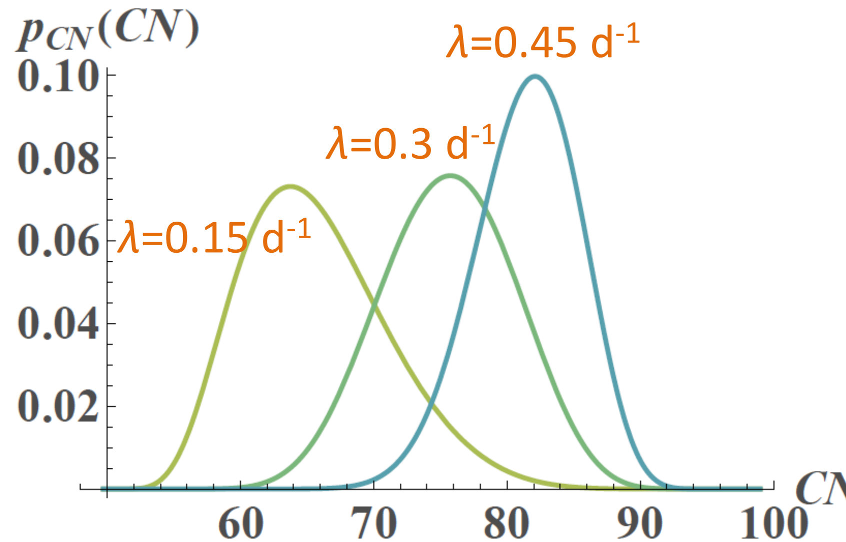
α=1.5 cm

w_0=24 cm (max. basin retention)

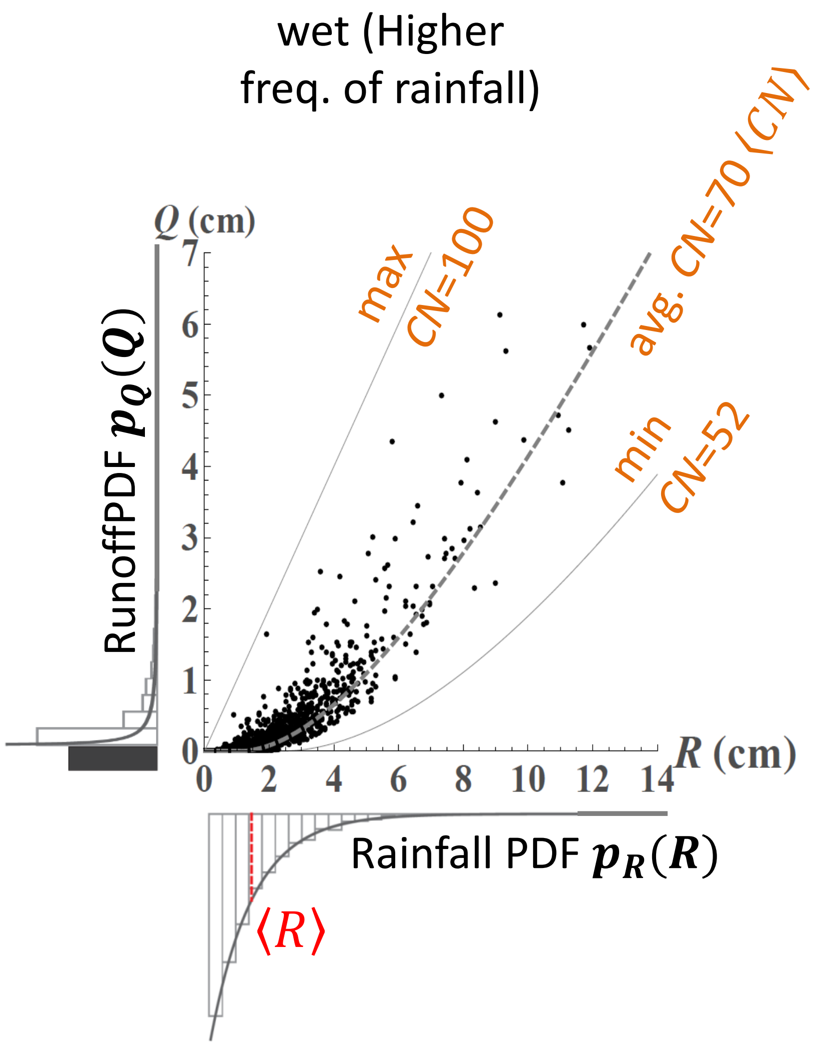


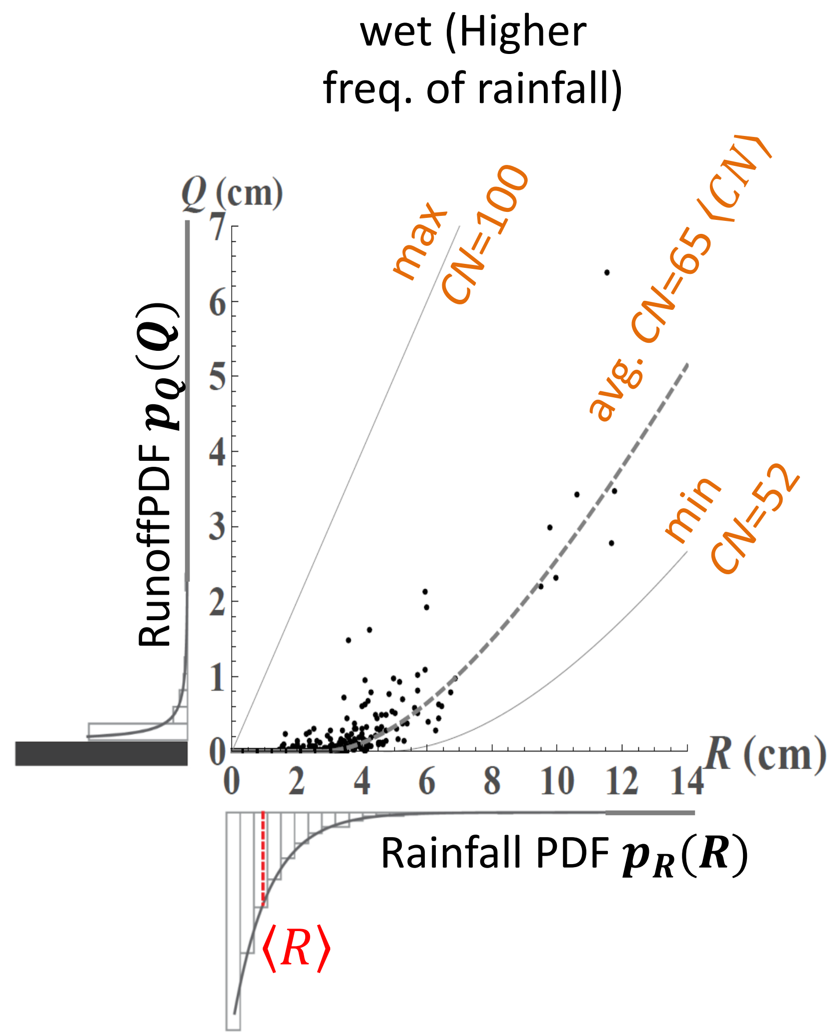
PDF of curve number (*CN*)

$$CN = \frac{1000}{10 + \frac{B\ (\text{mm})}{25.4}}$$

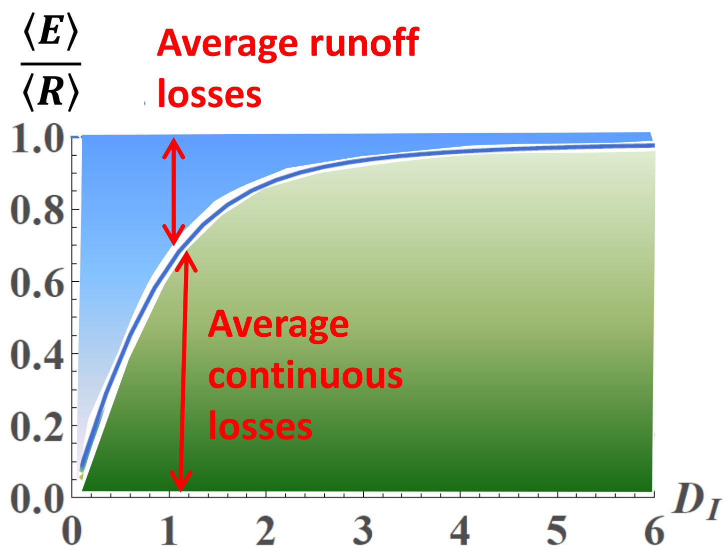


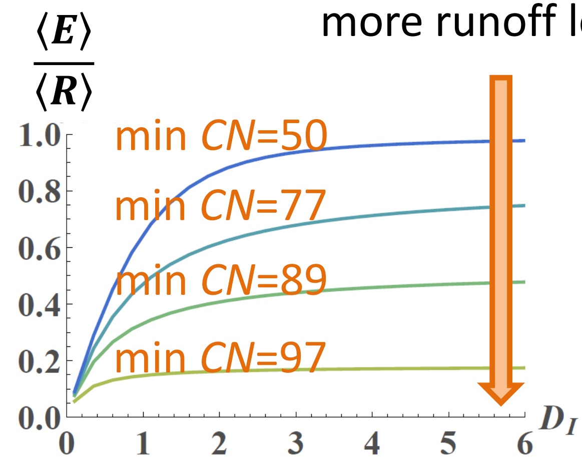
Individual realizations





Budyko-type curve[1] for average hydrologic balance





$$\frac{\langle \boldsymbol{E} \rangle}{\langle \boldsymbol{R} \rangle} = \frac{Average\ cont.loss = \langle 1-b \rangle \eta}{Avg.rainfall = \lambda\alpha} \quad Dryness\ Index, \boldsymbol{D_I} = \frac{Potential\ loss = \eta}{Avg.rainfall = \lambda\alpha}$$

λ Average frequency of input events
α Average input amount (normalized)
η is the normalized max. continuous loss rate

[1] Rodriguez-Iturbe, I. and Porporato, A. (2004). *Ecohydrology of Water-Controlled Ecosystems, p.* 55-56.

Conclusions

- We combined: - stochastic methods of soil water balance
  - Empirical runoff formulation of SCS – CN method used by the SWAT model
- Result: a parsimonious but realistic representation of the water balance, which improves ecohydrological models and determines consistently the retention parameter
- This allows **long term analyses** of the effects of changes in PET, freq. of rainfall, etc. on the soil water balance
- Easily coupled to other processes (water stress, biogeochemistry) to perform **probabilistic risk analysis and uncertainty quantification when planning for sustainable use of soil and water resources**

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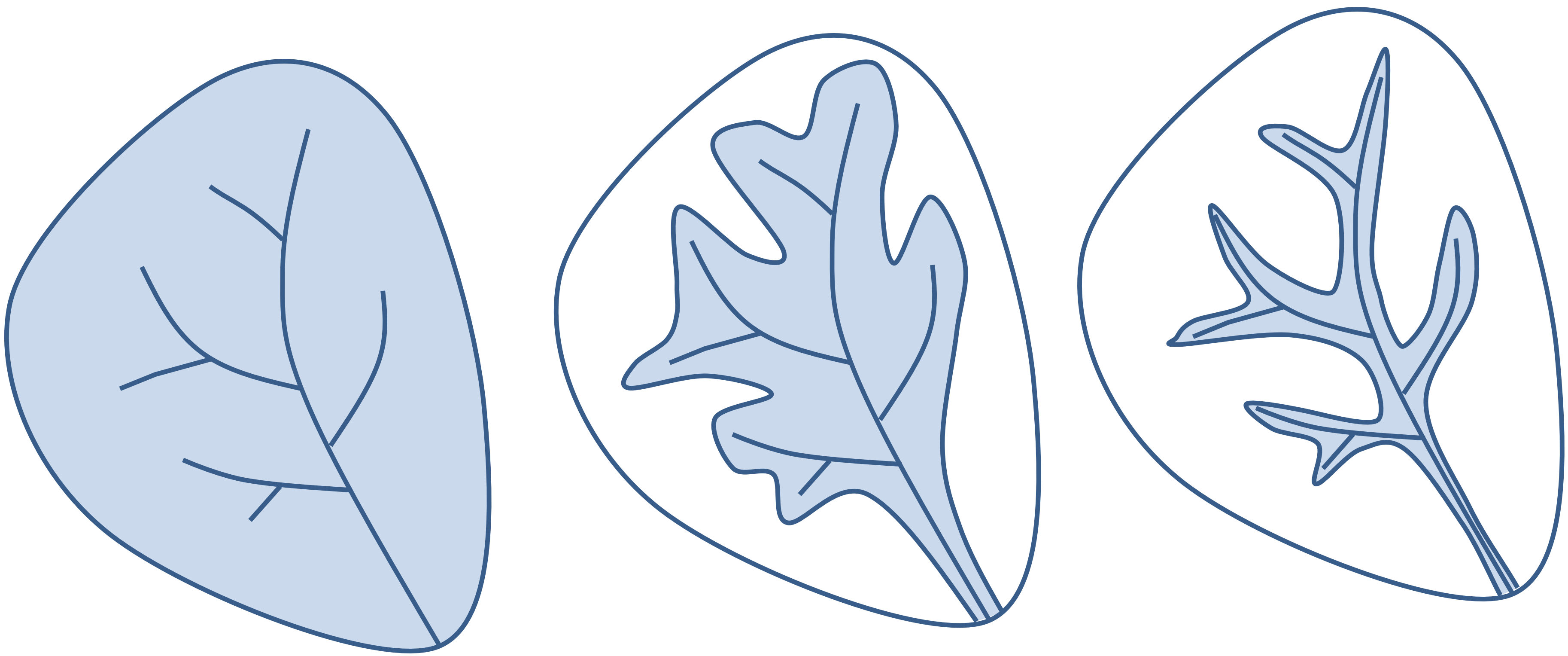


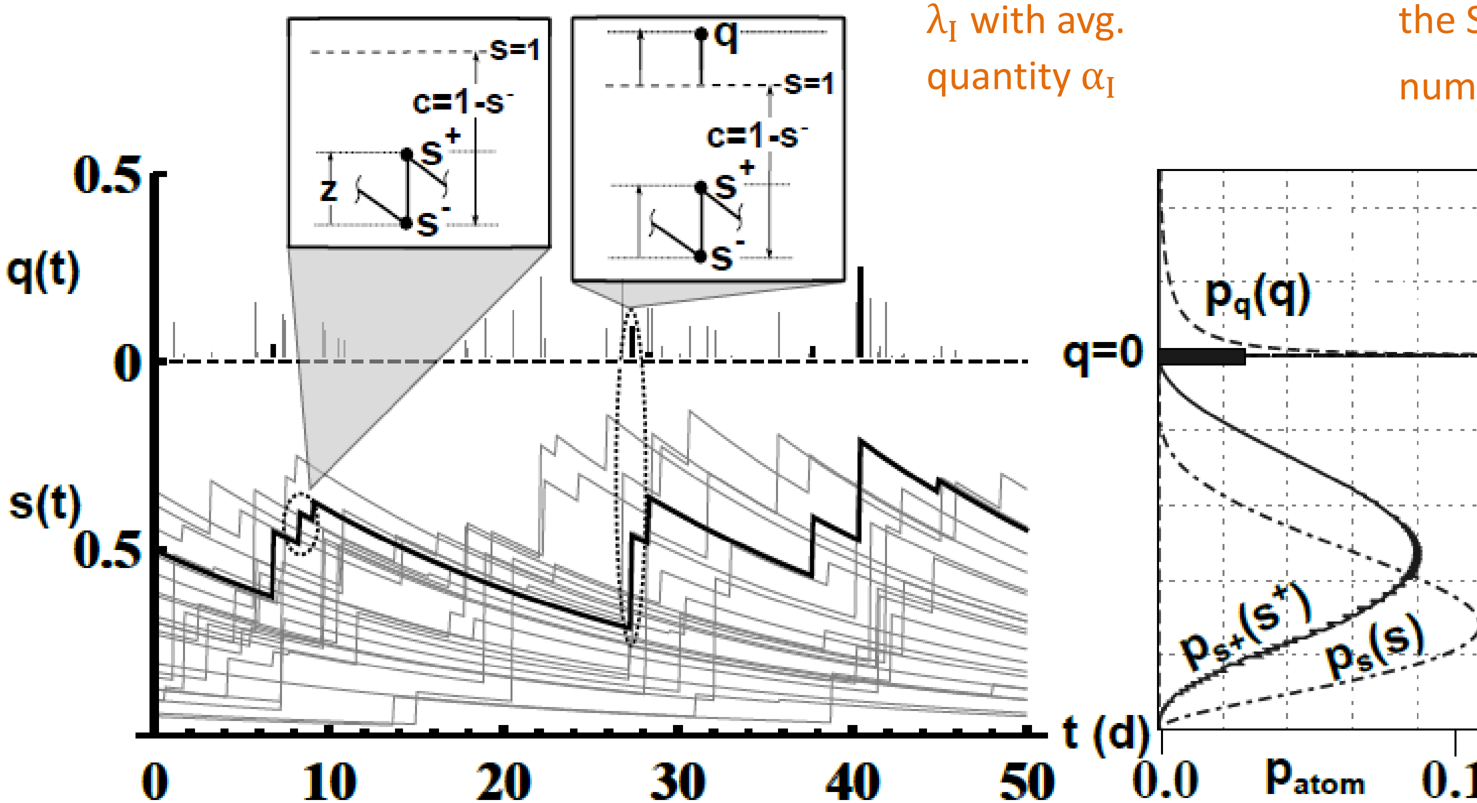
Conceptual probabilistic framework

Total storage depth $w_0 \dfrac{ds(t)}{dt} = I(t) - f_L[s(t)] - Q[s(t)]$

Rate for continuous water loss

Inputs (random pulses) at freq. λ_I with avg. quantity α_I

Runoff (pulses) partitioned by the SCS curve number



Probabilistic description of the storage of a simple reservoir component

Continuous loss term $f_L[s(t)]$

Loss from the jump

Infiltration before initial abstraction

$$\frac{\partial}{\partial t} p_b(b;t) = \frac{\partial}{\partial b}[f_L(b)p_b(b;t)] - \lambda p_b(b;t) + \lambda \int^{s} p_z(u-b)p_s(u;t)du$$
$$+\lambda \int \frac{u^2}{\big(u-((u-b)-\beta u)\big)^2} p_z\left(\beta u + \frac{u\big((u-b)-\beta u\big)}{u-\big((u-b)-\beta u\big)}\right) p_b(b;t)du$$

Probability density function (PDF) of normalized basin retention, *b*

Infiltration after initial abstraction