Integration of a pseudo 3D finite element ground water model with SWAT

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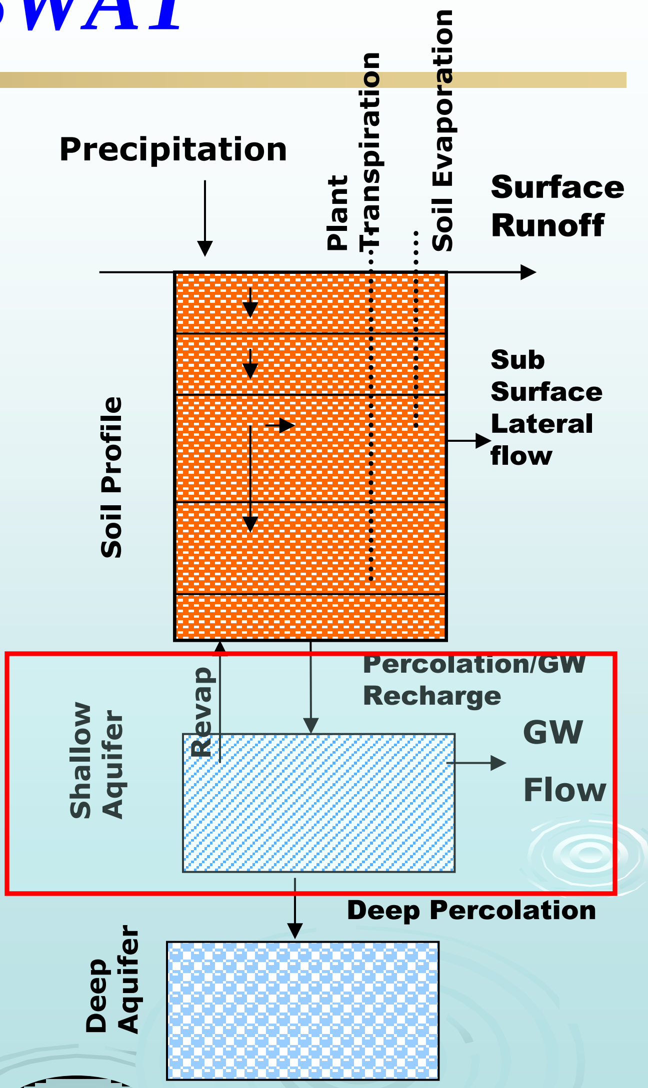
Outline

- Ground water model in SWAT
  - Assumptions and Limitations
- Pseudo 3D FEM
  - Assumptions and Limitations
- Preliminary assessment of the model
  - Upper Son Basin, Ganges, India

Groundwater Modelling in SWAT

SWAT simulates two types of aquifer for each sub-basin/HRU :

- Shallow Aquifer
- Deep Aquifer

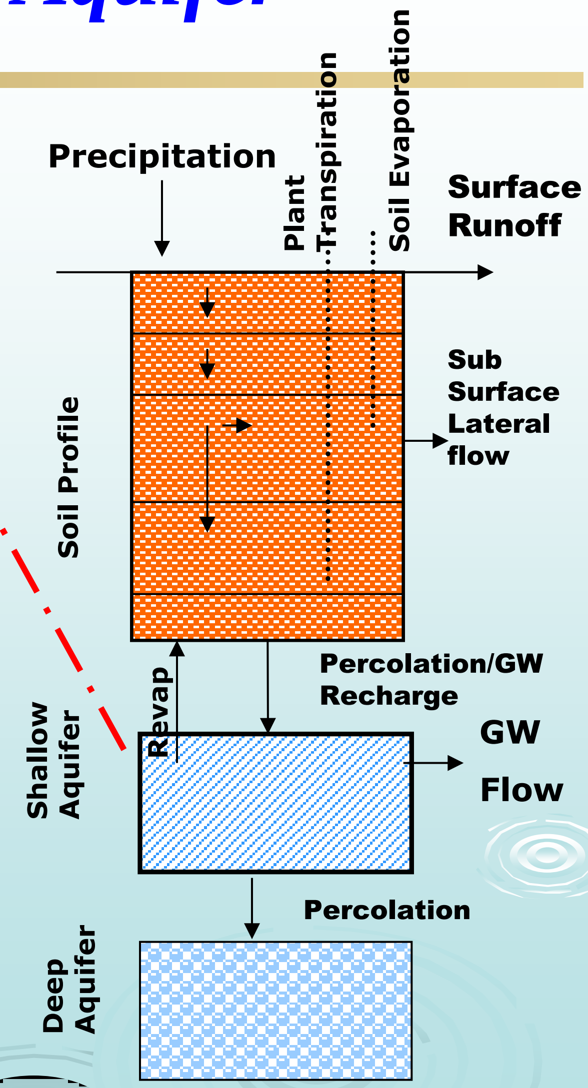


Water balance in Shallow Aquifer

$$aq_{sh,i} = aq_{sh,i-1} + w_{rchrg,sh} - Q_{gw} - w_{revap} - w_{pump,sh}$$

Where

- aqsh,i = amount of water stored in the shallow aquifer on day i (mm of water)
- aqsh,i-1 = amount of water stored in the shallow aquifer on day i-1 (mm of water)
- wrchrg,sh = amount of recharge entering the shallow aquifer on day i (mm of water)
- Qgw = Groundwater flow, or base flow into main channel on day i (mm of water)
- w_{revap} = amount of water moving into the soil zone in response to water deficiencies in day i (mm of water)
- wpump,sh = amount of water removed from shallow aquifer by pumping on a day i (mm of water)



Computation of Recharge

Recharge calculated in both aquifers are

$$w_{rhrg,i} = (1- \exp[-1/\delta_{gw}]).w_{seep} + \exp[-1/\delta_{gw}].w_{rchrg,i-1}$$

Exponential decay weighting function to simulate the delay from percolation to recharge

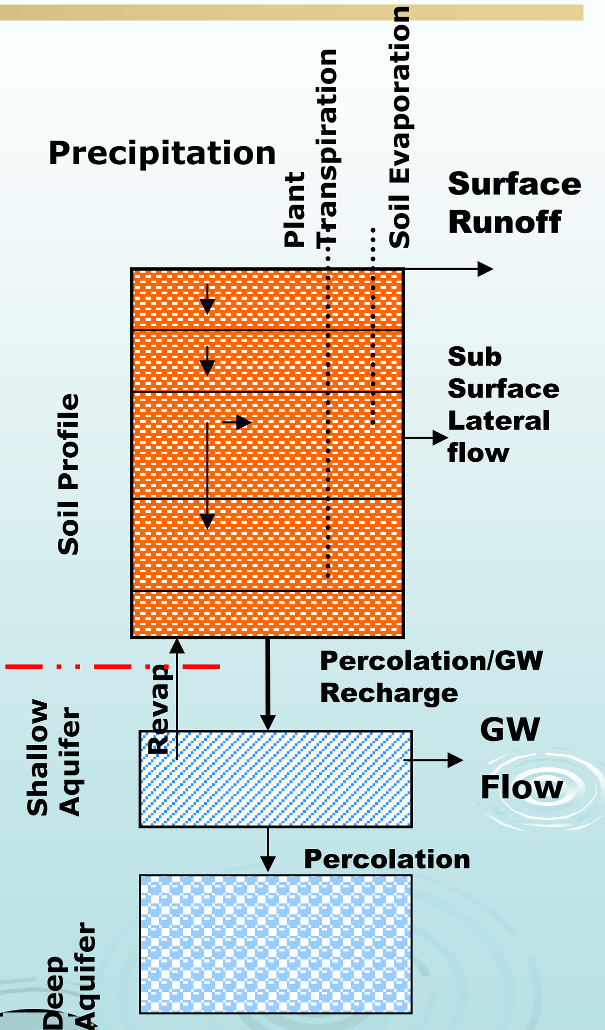
Where,

$w_{rhrg,i}$ = amount of recharge entering the aquifers on day i (mm of water)

δ_{gw} = delay time or drainage time of the overlying geologic formations (days)

W_{seep} = total amount of water exiting the bottom of the soil profile on day i (mm of water)

$w_{rchrg,i-1}$ = amount of recharge entering the aquifers on day i-1 (mm of water)

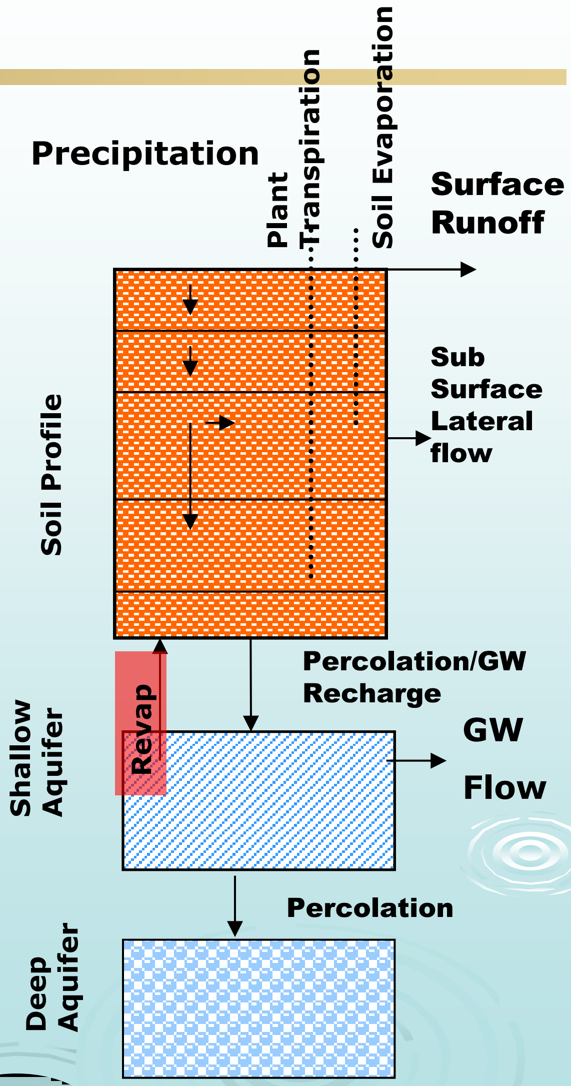


REVAP

- Maximum amount of water removed from the aquifer via REVAP

$$W_{revap,mx} = \beta_{rvp} .Ea$$

Capillary movement of water from shallow aquifer back to root zone



Where,

$W_{revap,mx}$ = maximum amount of water moving into soil zone in response to water deficiencies (mm of water)

β_{rev} = revap coefficient

Ea = actual evapotransipiration

Groundwater flow or Base flow

Base flow occur only if the amount of water stored in the shallow aquifer exceeds a threshold value specified by user , $aq_{shthr,q}$

The baseflow rate is modelled using Hooghoudt's equation

Hooghoudt's equation

❖ Hooghoudt's found the equation for steady state response of groundwater flow to recharge is

$$Q_{gw} = \frac{8000.K_{sat}}{L_{gw}^2} h_{wtbl}$$

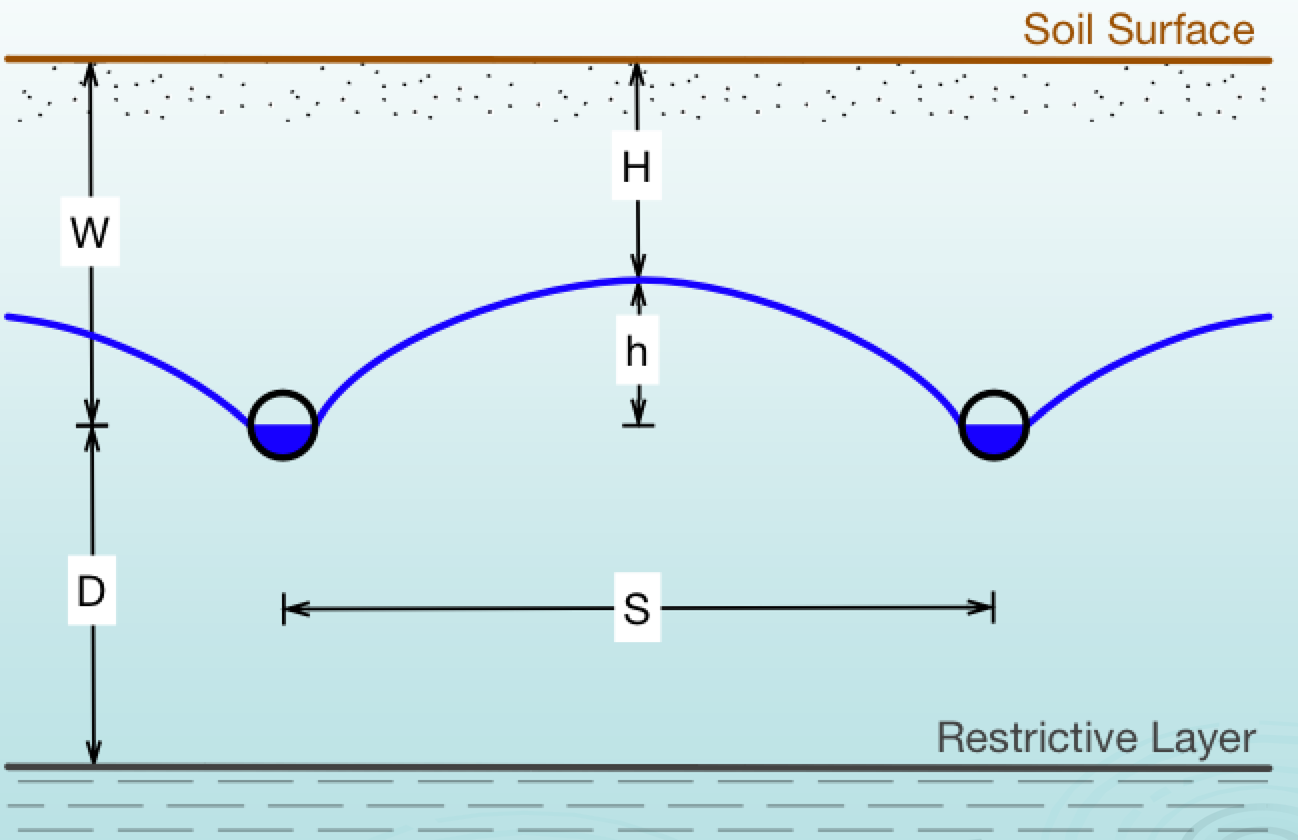
Where,
Q_{gw} = Groundwater flow or base flow, into the main channel on day i (mm of water)

K_{sat} = Hydraulic conductivity of the aquifer (mm/day)

L_{Gw} =Distance from the ridge or sub basin divide for the groundwater system to the main channel (m)

h_{wtbl} =Water table height (m)

Drain Spacing design



Source; http://climate.sdstate.edu/water/drainspacingcalculatordocumentation.html

Assumptions by Hooghoudt's

- Soil is homogeneous
- Darcy's law is valid for the flow
- The hydraulic gradient at any point is equal to the slope of the water table above that point ($I = dy/dx$) and the water flows horizontally

Limitations

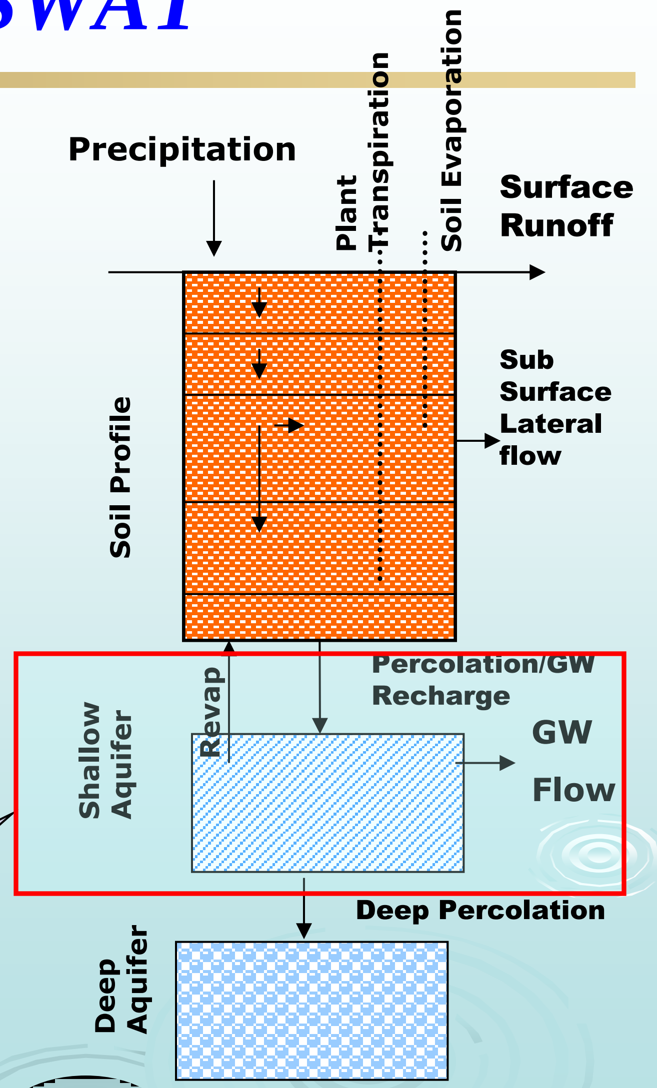
- 1D model
- Steady state assumption
- No interaction among HRU’s

Groundwater Modelling in SWAT

SWAT simulates two types of aquifer for each sub-basin/HRU :

- Shallow Aquifer
- Deep Aquifer

Model this in FEM



SWAT – FEM Coupling

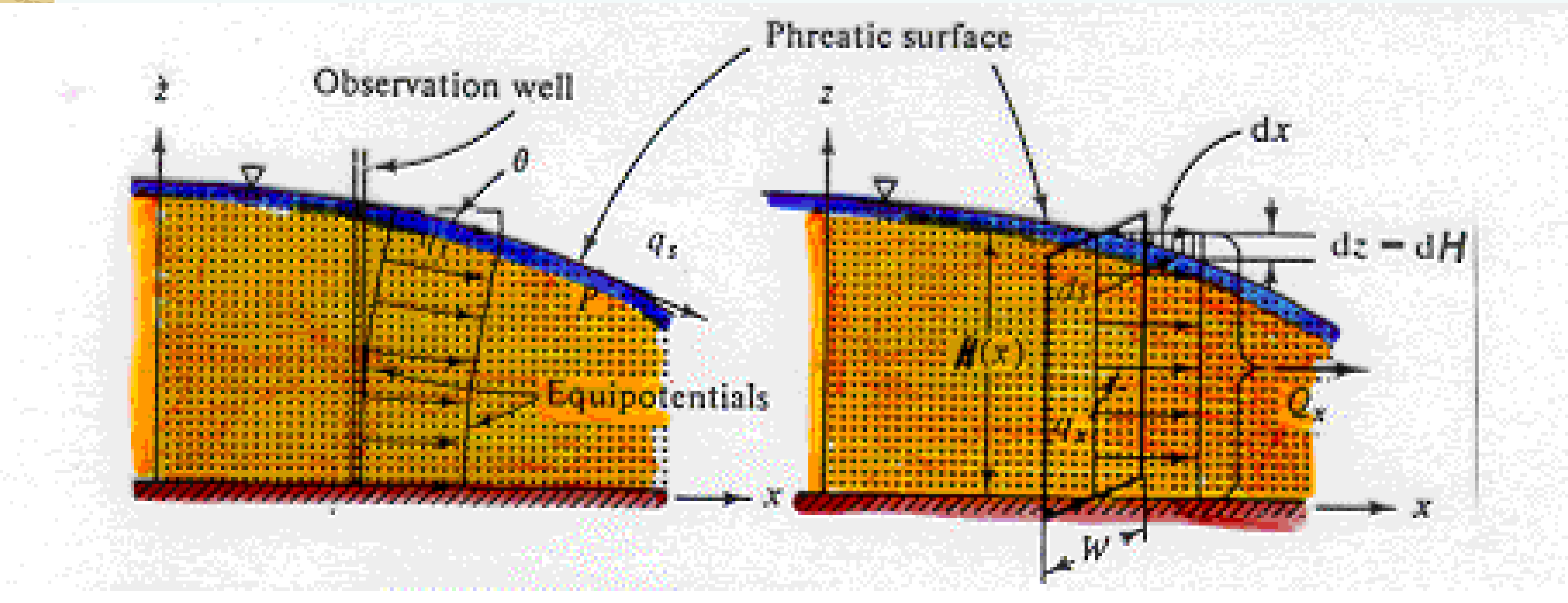
- SWAT model was coupled with a pseudo 3D finite element ground water model developed by Narasimhan and Sri Ranjan. (2000)

$$\frac{\delta}{\delta x}\left((k(x,y)\mathrm{h}\ \frac{\delta h}{\delta x}\right) + \frac{\delta}{\delta y}\left((k(x,y)\mathrm{h}\ \frac{\delta h}{\delta y}\right) + \mathrm{Q}\ (\mathrm{x,y,t}) = \mathrm{Sy}\ \frac{\delta h}{\delta t}$$

Assumptions

- The flow is horizontal
  - Vertical hydraulic gradient is assumed to be zero – Dupuit's assumptions
- The aquifer formation is primarily horizontal
- The aquifer can be anisotropic and heterogeneous in X and Y direction

Dupuit's Assumption

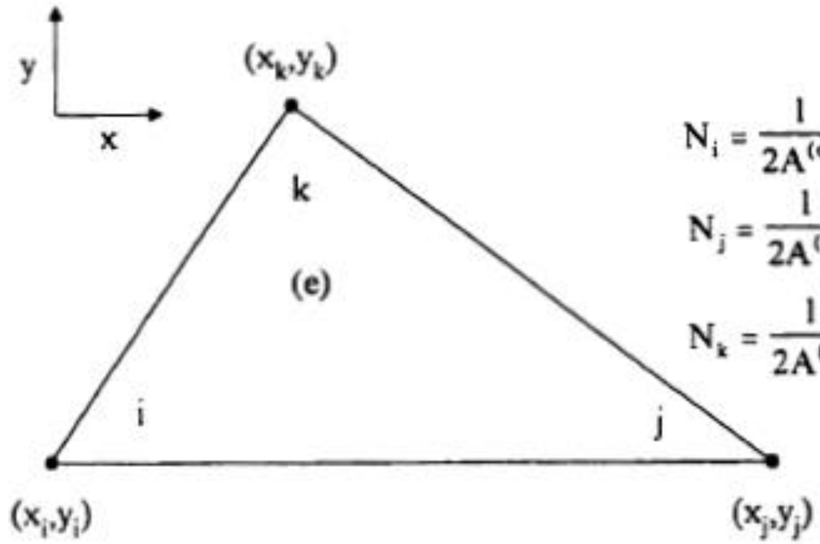


Source: http://www.interpore.org/reference_material/mgfc-course/mgfcaqtr.html

Finite Element Solution

- Galerkin's formulation
  - Selection of finite element shape
  - Trial function
    - Linear Vs. Non-linear
  - Formulation of element matrices
  - Finite difference formulation for the time derivative
  - Solving the system of equations using LU decompositions

Linear Triangular Finite Elements



$$N_i = \frac{1}{2A^{(e)}}(a_i + b_i x + c_i y)$$

$$N_j = \frac{1}{2A^{(e)}}(a_j + b_j x + c_j y)$$

$$N_k = \frac{1}{2A^{(e)}}(a_k + b_k x + c_k y)$$

where:

$$a_i = x_j^{(e)}y_k^{(e)} - x_k^{(e)}y_j^{(e)} \qquad a_j = x_k^{(e)}y_i^{(e)} - x_i^{(e)}y_k^{(e)} \qquad a_k = x_i^{(e)}y_j^{(e)} - x_j^{(e)}y_i^{(e)}$$

$$b_i = y_j^{(e)} - y_k^{(e)} \qquad b_j = y_k^{(e)} - y_i^{(e)} \qquad b_k = y_i^{(e)} - y_j^{(e)}$$

$$c_i = x_k^{(e)} - x_j^{(e)} \qquad c_j = x_i^{(e)} - x_k^{(e)} \qquad c_k = x_j^{(e)} - x_i^{(e)}$$

$A^{(e)}$ = Area of element

$$A^{(e)} = \frac{1}{2}\begin{vmatrix} 1 & x_i^{(e)} & y_i^{(e)} \\ 1 & x_j^{(e)} & y_j^{(e)} \\ 1 & x_k^{(e)} & y_k^{(e)} \end{vmatrix}$$

Formulation of Element Matrices

$$[K^{(e)}] = \int_A \left(D_{dx_i} \frac{\partial [N]^T}{\partial x} \frac{\partial [N]}{\partial x} + D_{dy_i} \frac{\partial [N]^T}{\partial y} \frac{\partial [N]}{\partial y} \right) dA + \int_A [N]^T \left(V_{tx_i} \frac{\partial [N]}{\partial x} + V_{ty_i} \frac{\partial [N]}{\partial y} \right) dA \qquad (4.11)$$

Solving Eq.4.11 using triangular elements gives:

$$[K^{(e)}] = \frac{D^{\bullet}_{dx}}{4A^{(e)}} \begin{bmatrix} b_i^2 & b_i b_j & b_i b_k \\ b_j b_i & b_j^2 & b_j b_k \\ b_k b_i & b_k b_j & b_k^2 \end{bmatrix} + \frac{D^{\bullet}_{dy}}{4A^{(e)}} \begin{bmatrix} c_i^2 & c_i c_j & c_i c_k \\ c_j c_i & c_j^2 & c_j c_k \\ c_k c_i & c_k c_j & c_k^2 \end{bmatrix} + \frac{V_{tx}}{6} \begin{bmatrix} b_i & b_j & b_k \\ b_i & b_j & b_k \\ b_i & b_j & b_k \end{bmatrix} + \frac{V_{ty}}{6} \begin{bmatrix} c_i & c_j & c_k \\ c_i & c_j & c_k \\ c_i & c_j & c_k \end{bmatrix} \qquad (4.12)$$

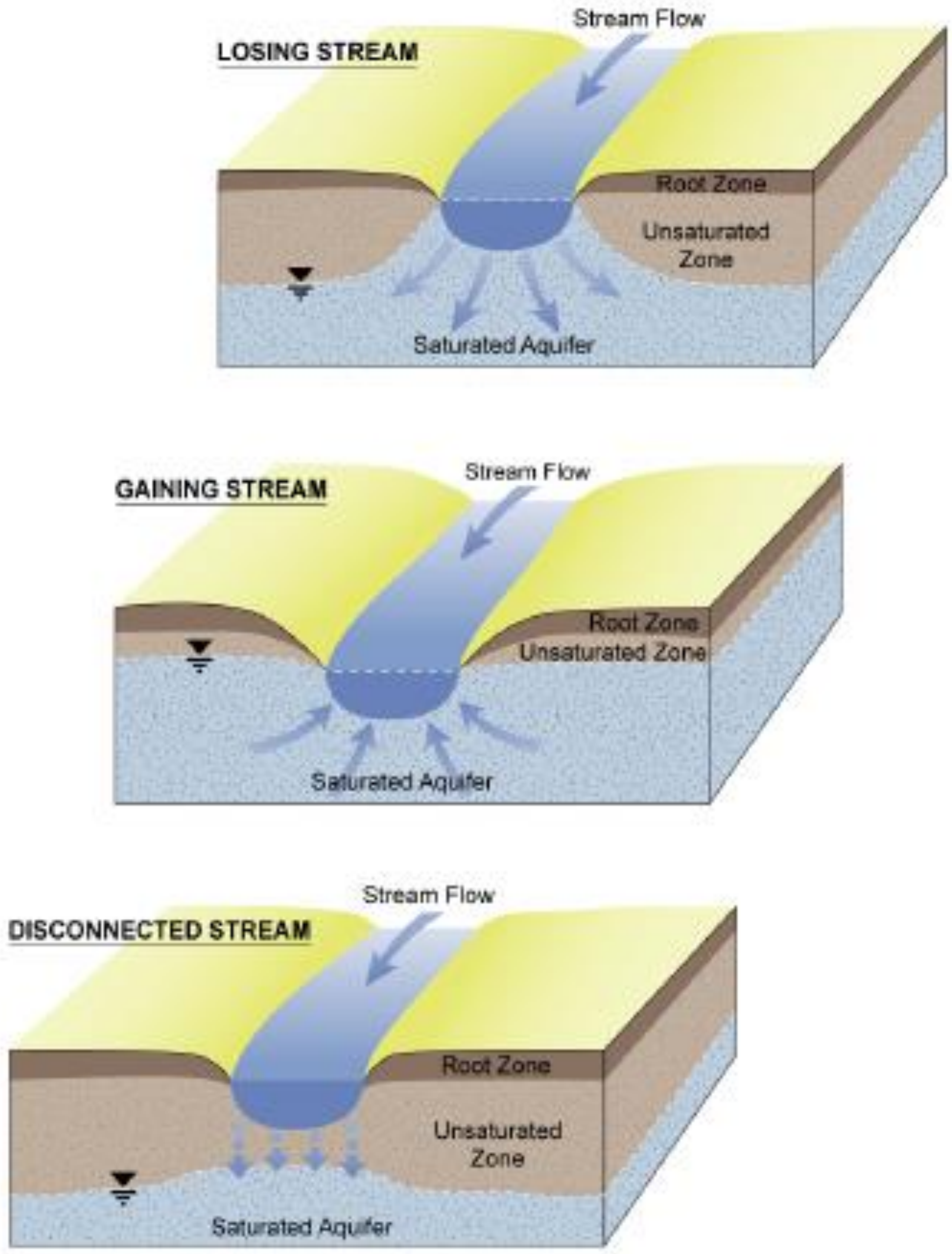
$$[C^{(e)}] = \int_A R_d \, [N]^T [N] dA$$

Solving Eq.4.15 using lumped formulation for triangular elements gives:

$$[C^{(e)}] = \frac{A^{(e)}}{3} R_{d_i} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Boundary conditions

- Time varying Flux boundary
  - Percolation from soil layer
- Source or Sink
  - Ground water pumping, REVAP and deep percolation
- Time varying head boundaries
  - Reservoirs
- Conditional time varying head boundaries
  - Connected/Disconnected river



Source: IWFM theoretical documentation

Finite difference solution in time

$$([C]+\theta\Delta t[K])\{C\}^{(e)}_{t+\Delta t} = ([C]-(1-\theta)\Delta t[K])\{C\}^{(e)}_{t} + \Delta t((1-\theta)\{F\}_t + \theta\{F\}_{t+1})$$

Eulers Backward difference method θ = 1

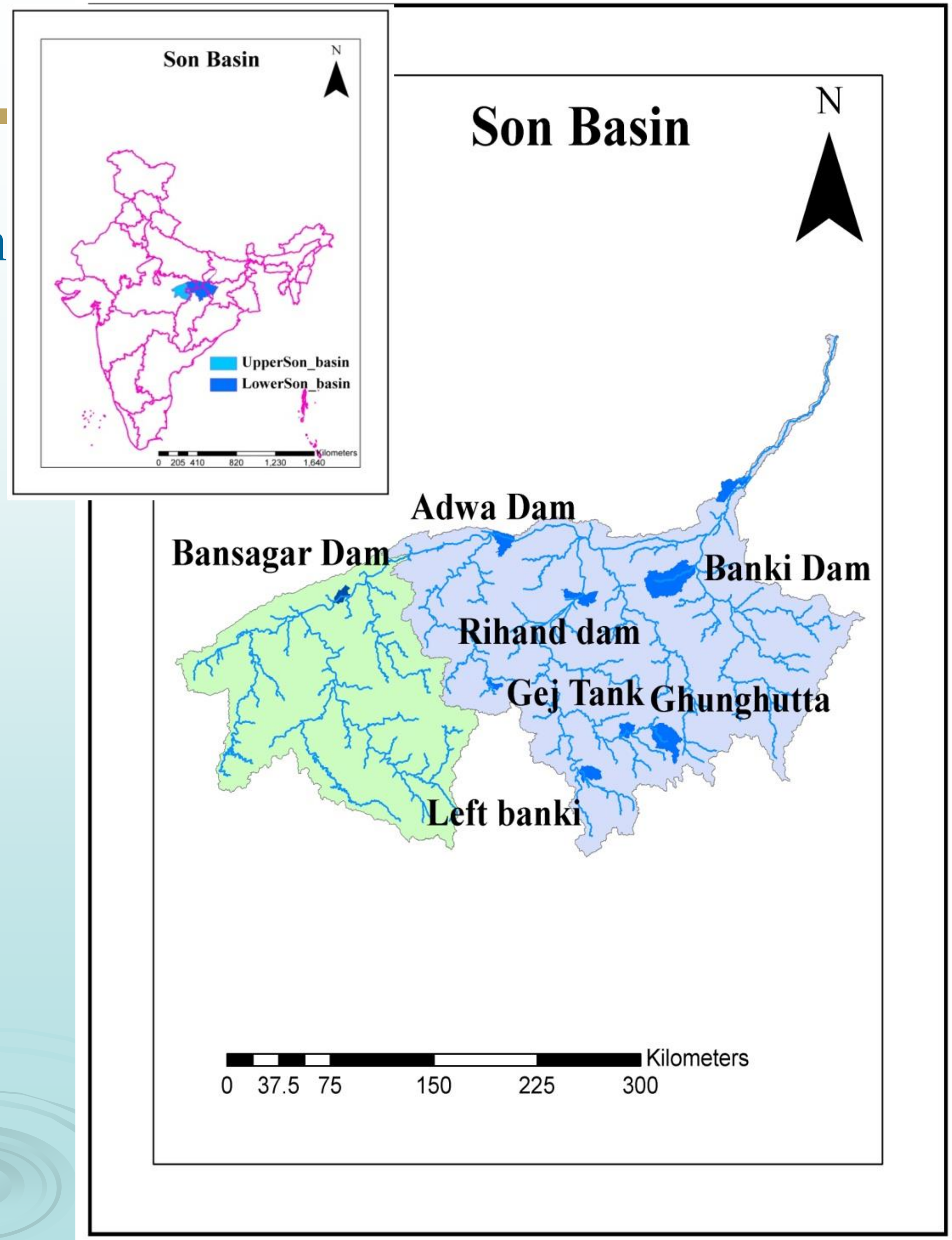
$$\left([K]+\frac{[C]}{\Delta t}\right)\{C\}^{(e)}_{t+1} = \frac{[C]}{\Delta t}\{C\}^{(e)}_{t} + \{F\}_{t+1}$$

Inputs For Coupling

- SWAT Model :: HRU == FEM Model :: Element
- HRU – ELE conversion (Percolation & Base flow)
  - Base flow when Avg ht. of water in element > Avg ht. of any node in the element
- Reach – Node conversion (Gaining, Loosing Stream, connected/disconnected stream)
- Reservoir – ELE conversion (Dry Elements and Wet Elements : based on the water level)

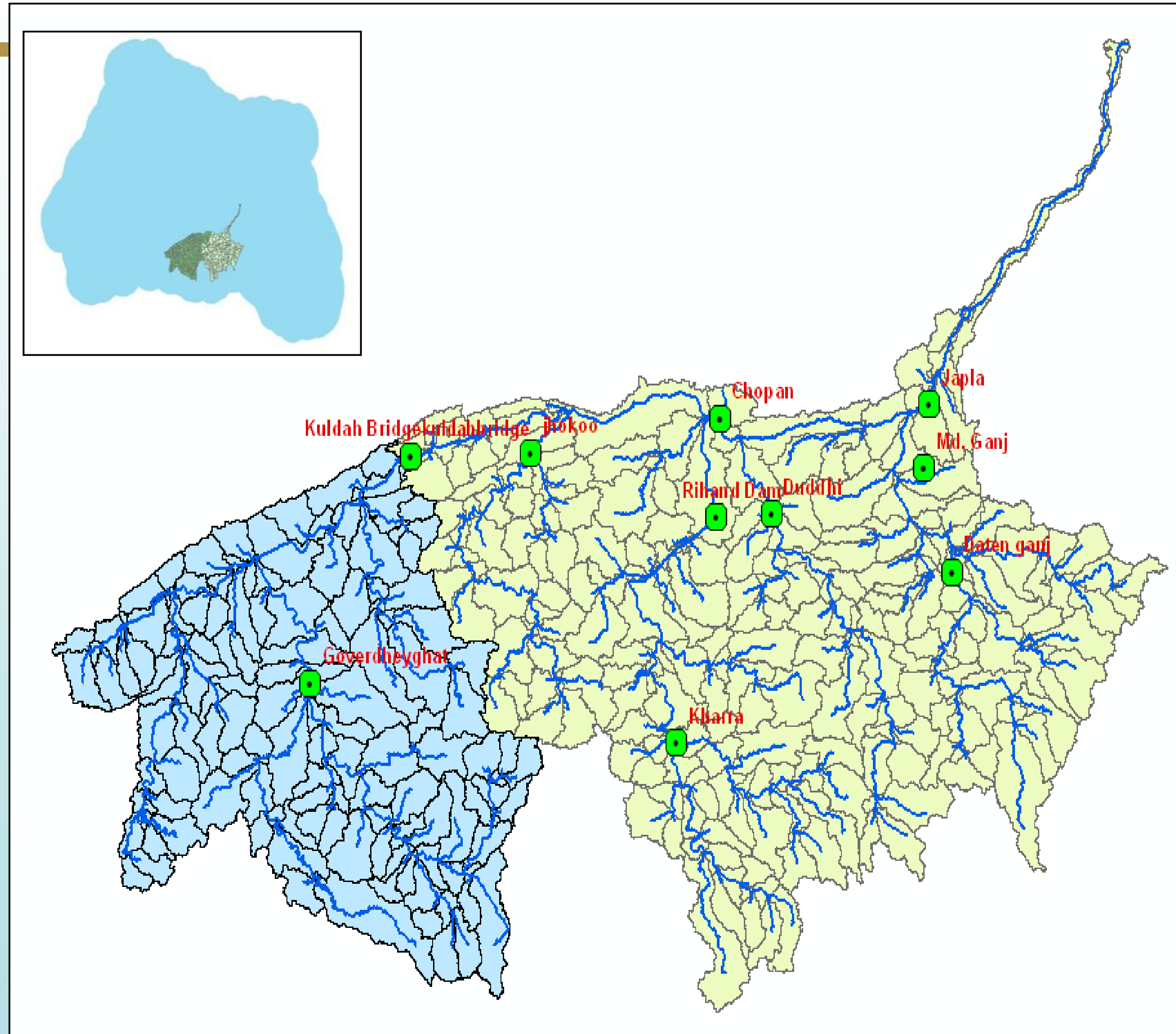
Study Area

- Son Basin in Ganga river
- Originates near Amarkantak in M.P. and flows north-north west
- Catchment area – 67,842 sq.km.
- Length – 784 km
- Tributaries – Rihand & North Koel
- Steep gradient – 0.35m to 0.55m / km



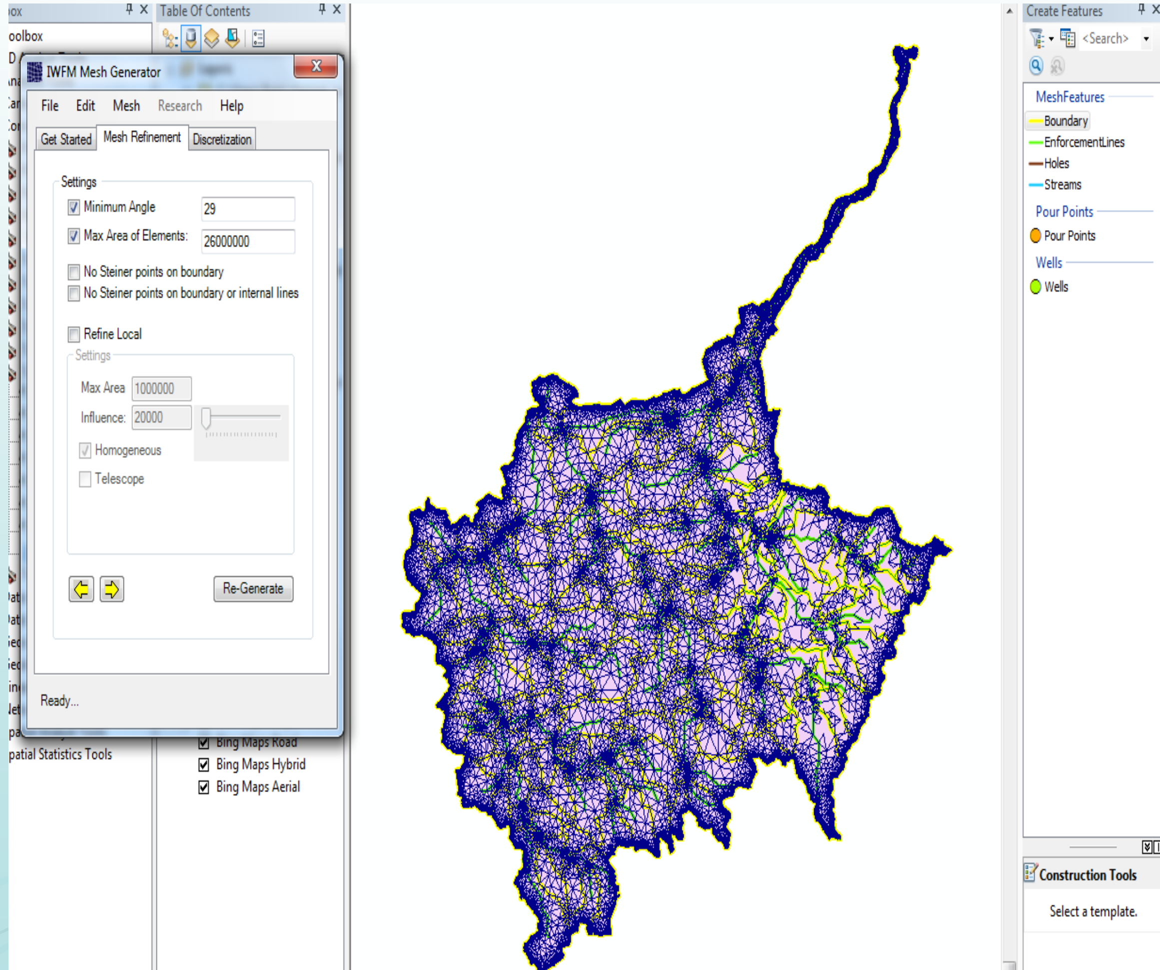
Watershed Delineation

- Flow-routing was done
- Upper Son
  - Sub basins – 143
  - HRUs - 599
- Lower Son
  - Sub basins – 248
  - HRUs - 1143



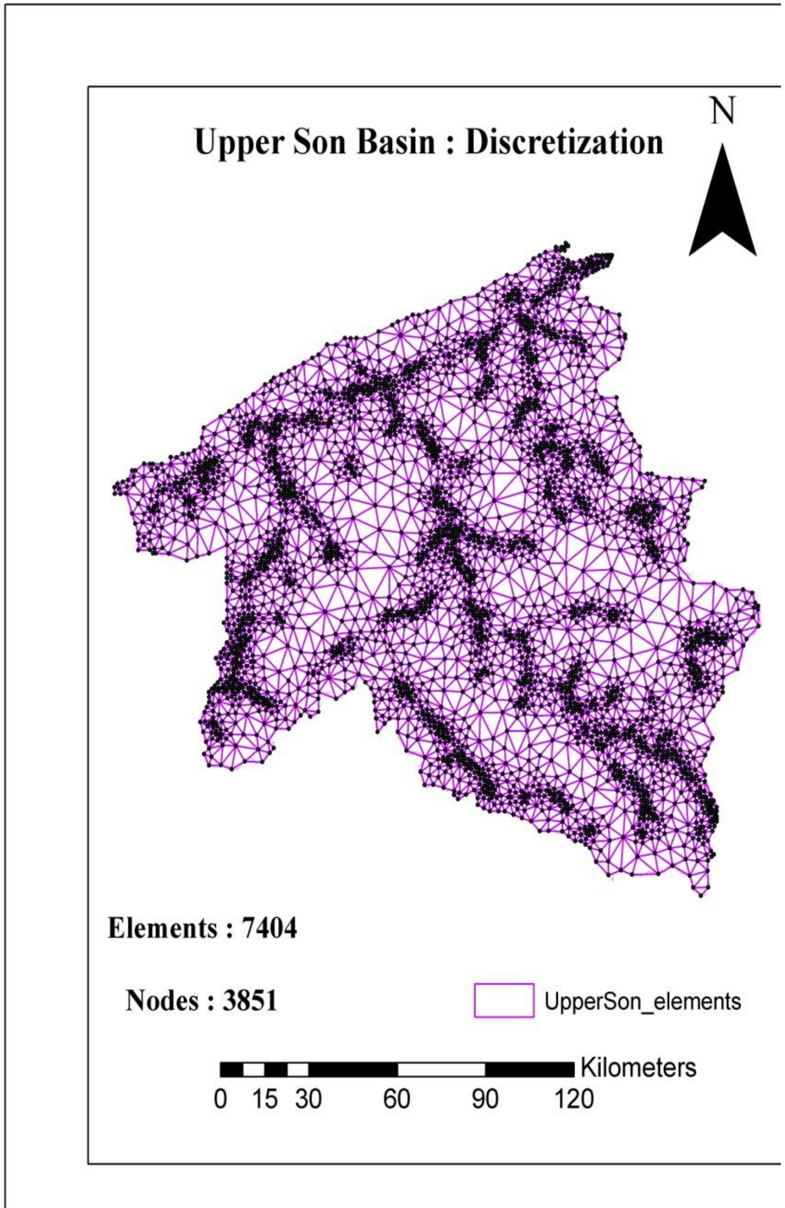
IWFM Mesh Generator Tool

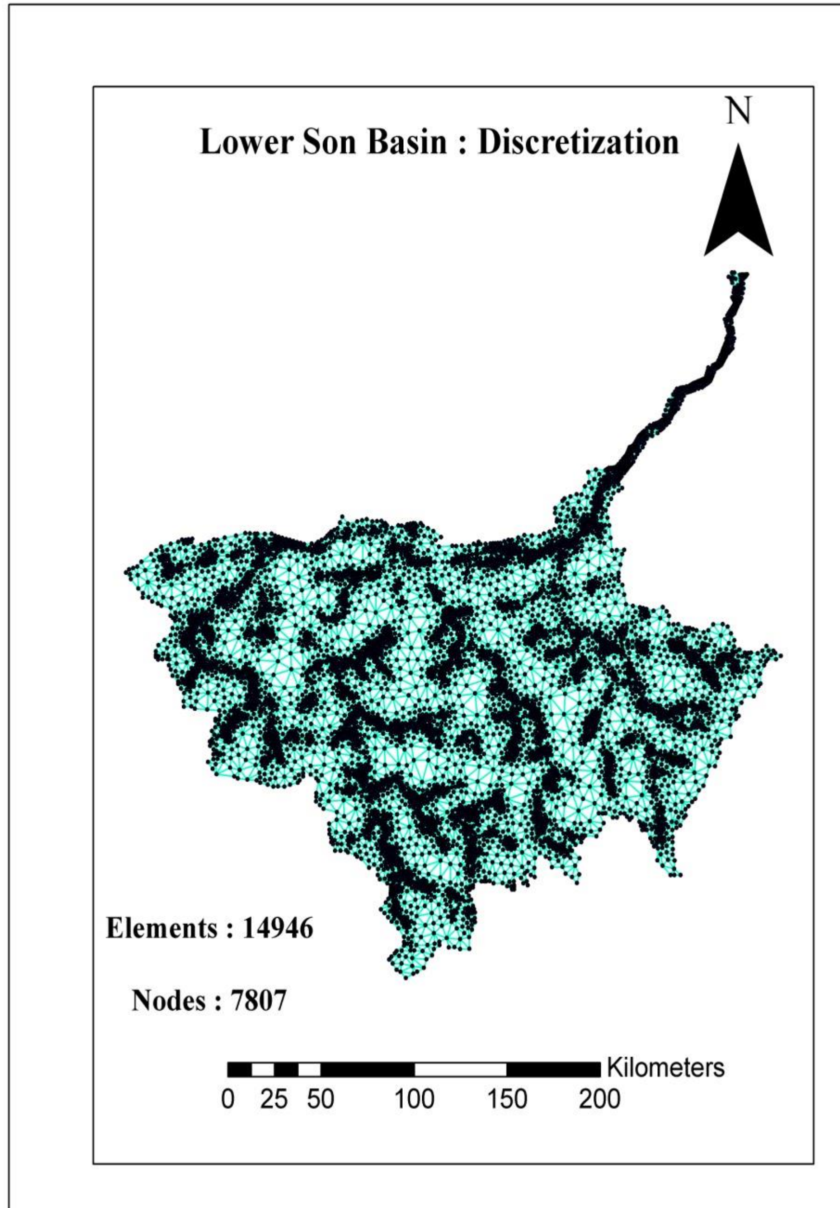
- Discretization of watershed
  - Uses Triangle to subdivide the study area into triangular elements
  - Inputs – Basin and stream boundary

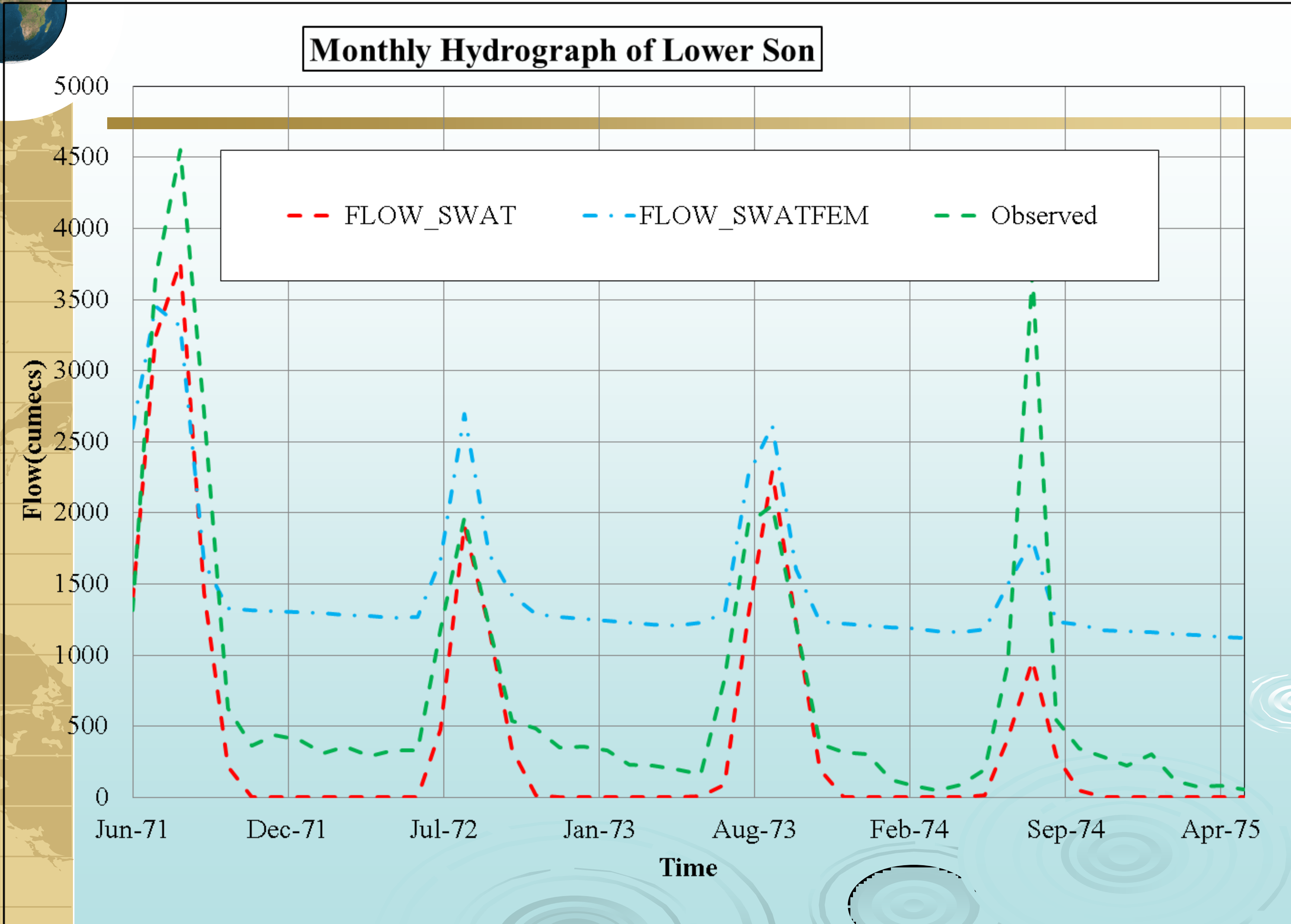


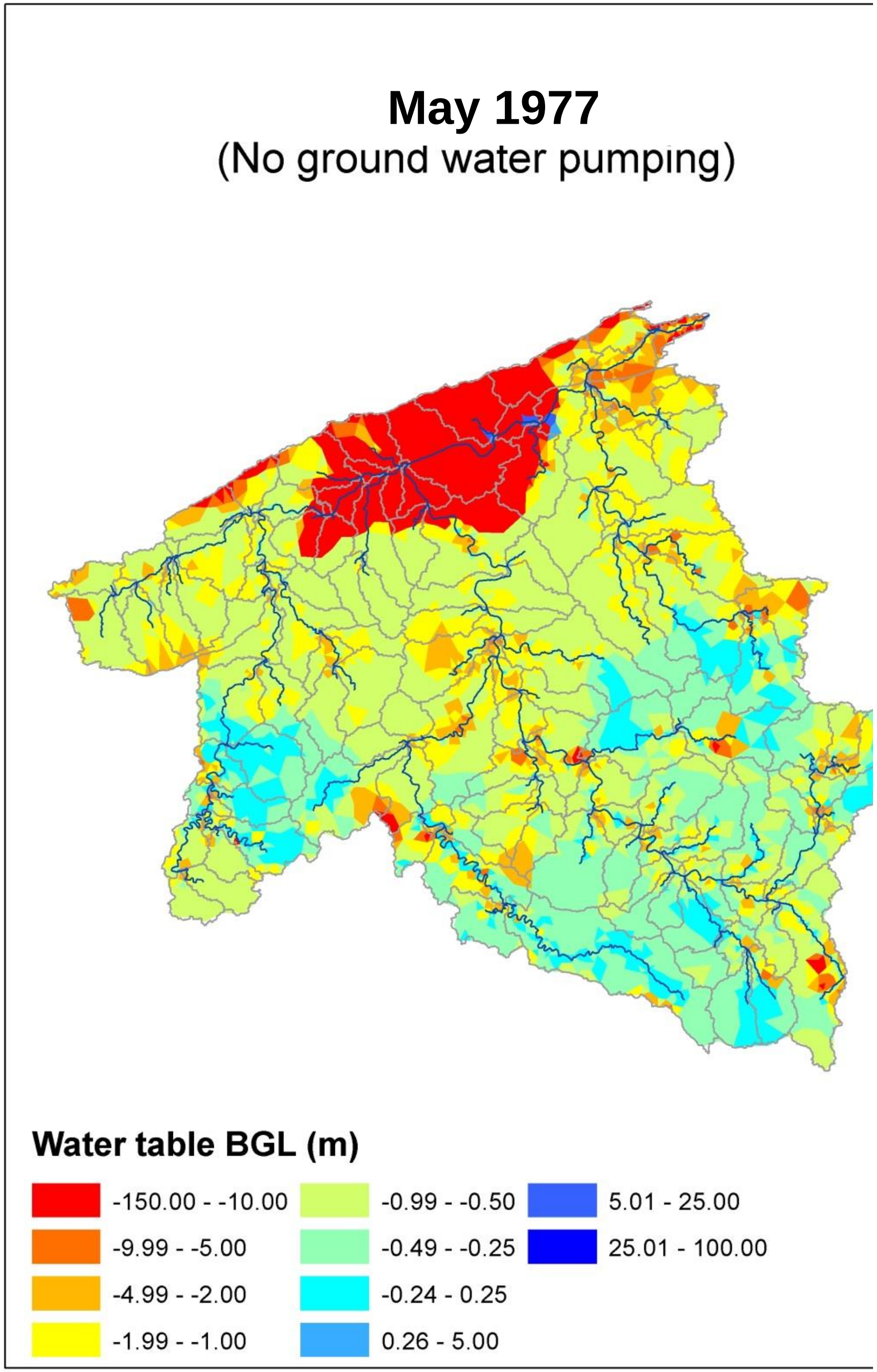
Integrated Hydrological Models Development Unit
Modeling Support Branch
Bay-Delta Office
February 2013

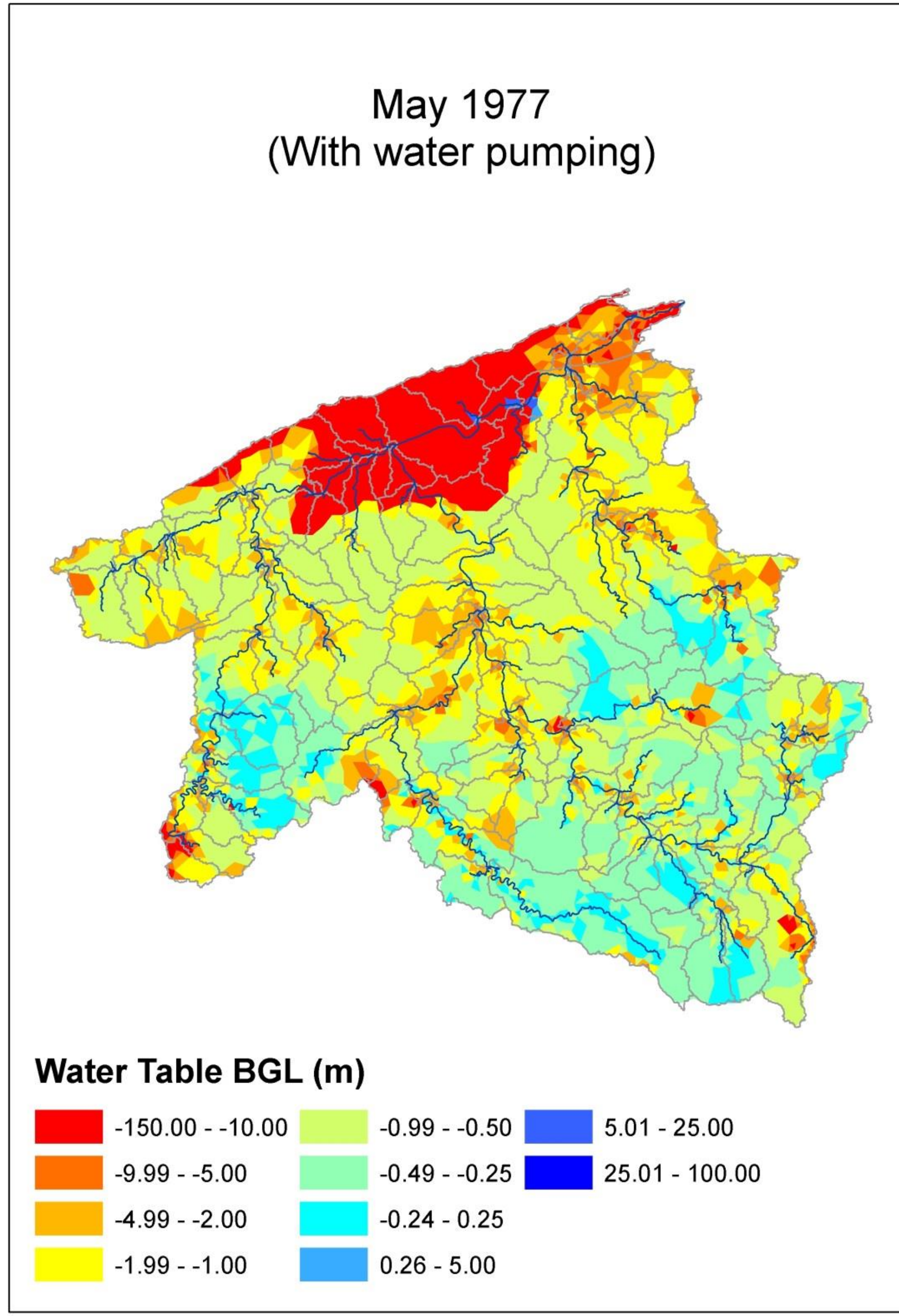
Discretized Upper and Lower Son Basin

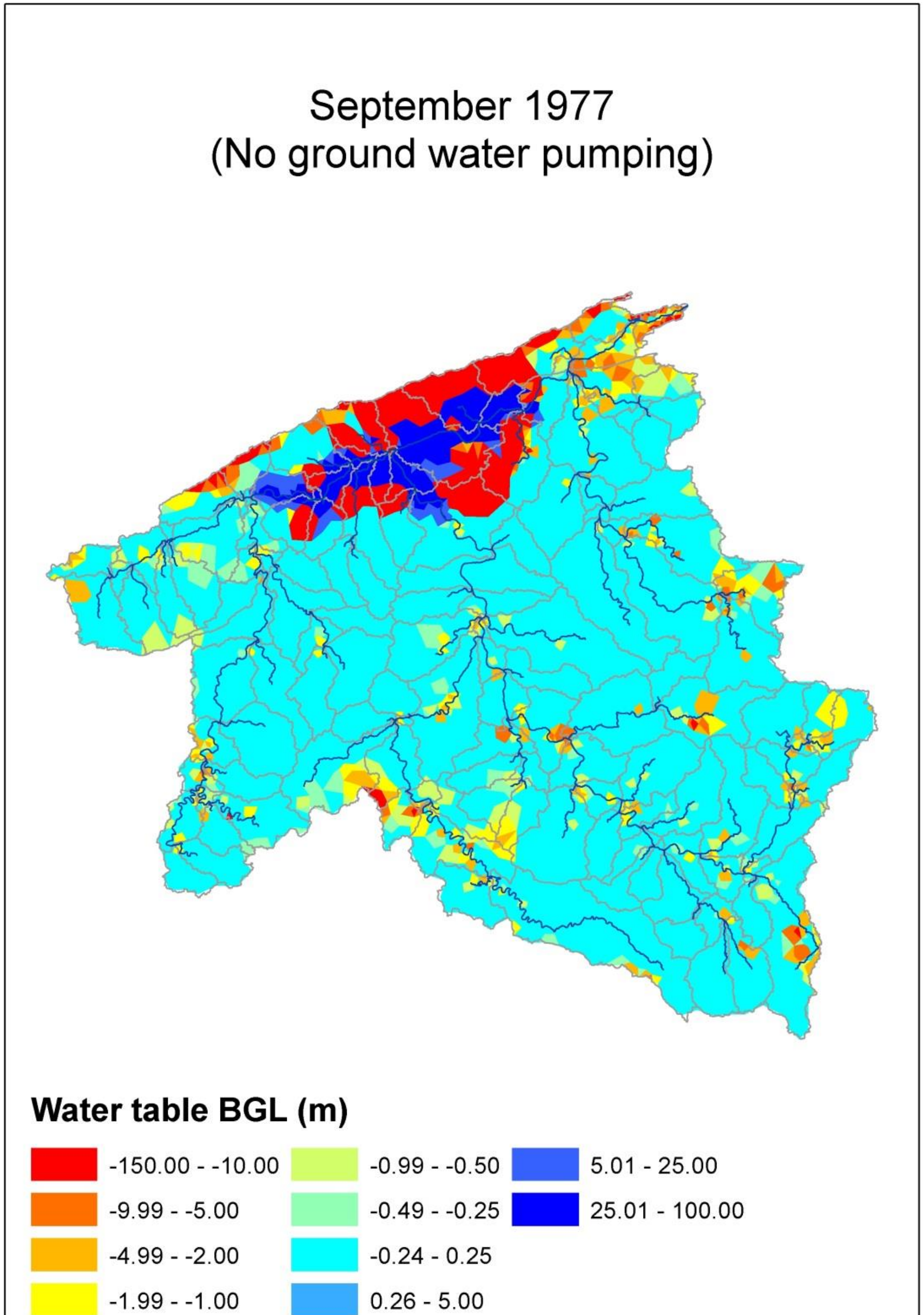





Monthly Hydrograph of Lower Son
FLOW_SWAT
FLOW_SWATFEM
Observed
Flow(cumecs)
5000
4500
4000
3500
3000
2500
2000
1500
1000
500
0
Jun-71
Dec-71
Jul-72
Jan-73
Aug-73
Feb-74
Sep-74
Apr-75
Time


May 1977
(No ground water pumping)
Water table BGL (m)
-150.00 - -10.00
-9.99 - -5.00
-4.99 - -2.00
-1.99 - -1.00
-0.99 - -0.50
-0.49 - -0.25
-0.24 - 0.25
0.26 - 5.00
5.01 - 25.00
25.01 - 100.00


May 1977
(With water pumping)
Water Table BGL (m)
-150.00 - -10.00
-9.99 - -5.00
-4.99 - -2.00
-1.99 - -1.00
-0.99 - -0.50
-0.49 - -0.25
-0.24 - 0.25
0.26 - 5.00
5.01 - 25.00
25.01 - 100.00


September 1977
(No ground water pumping)
Water table BGL (m)
-150.00 - -10.00
-9.99 - -5.00
-4.99 - -2.00
-1.99 - -1.00
-0.99 - -0.50
-0.49 - -0.25
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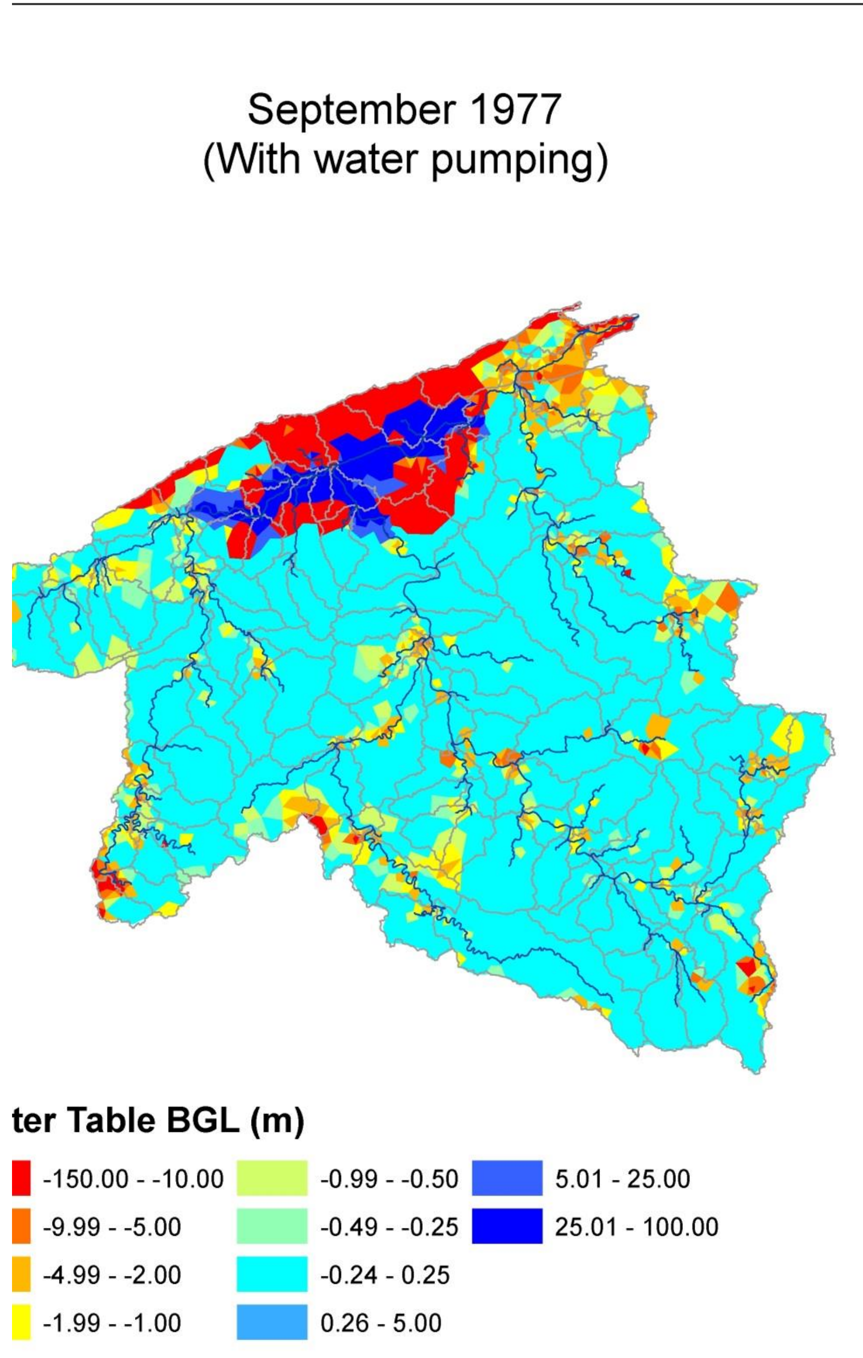
September 1977
(With water pumping)
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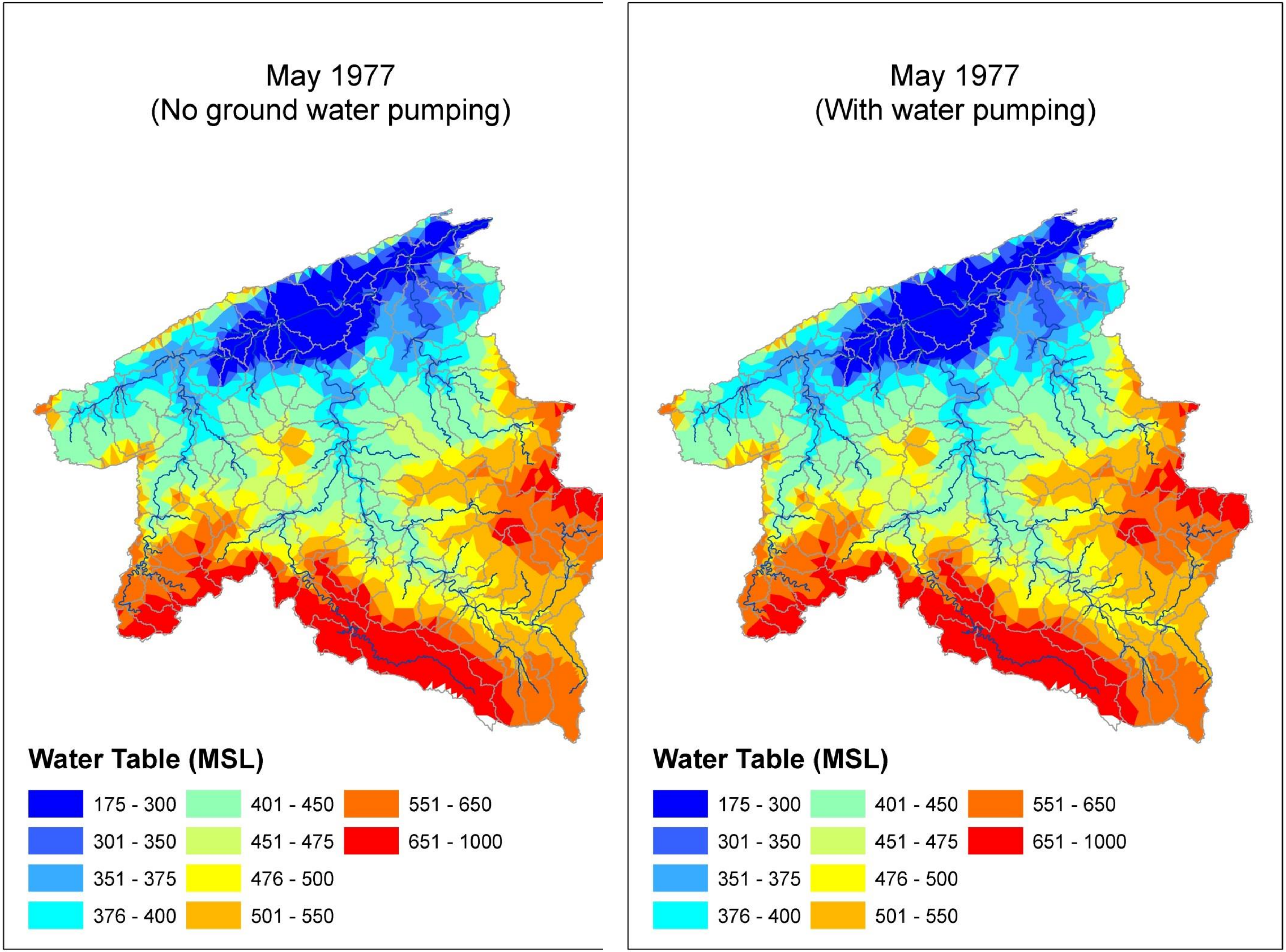
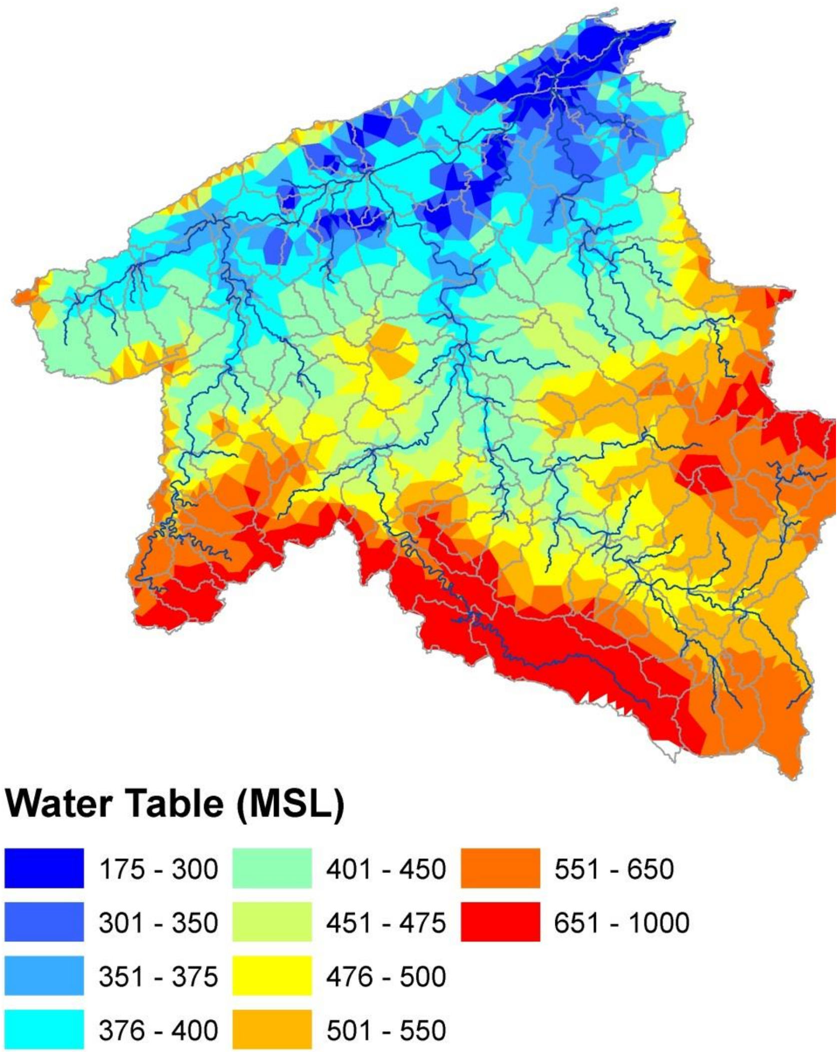
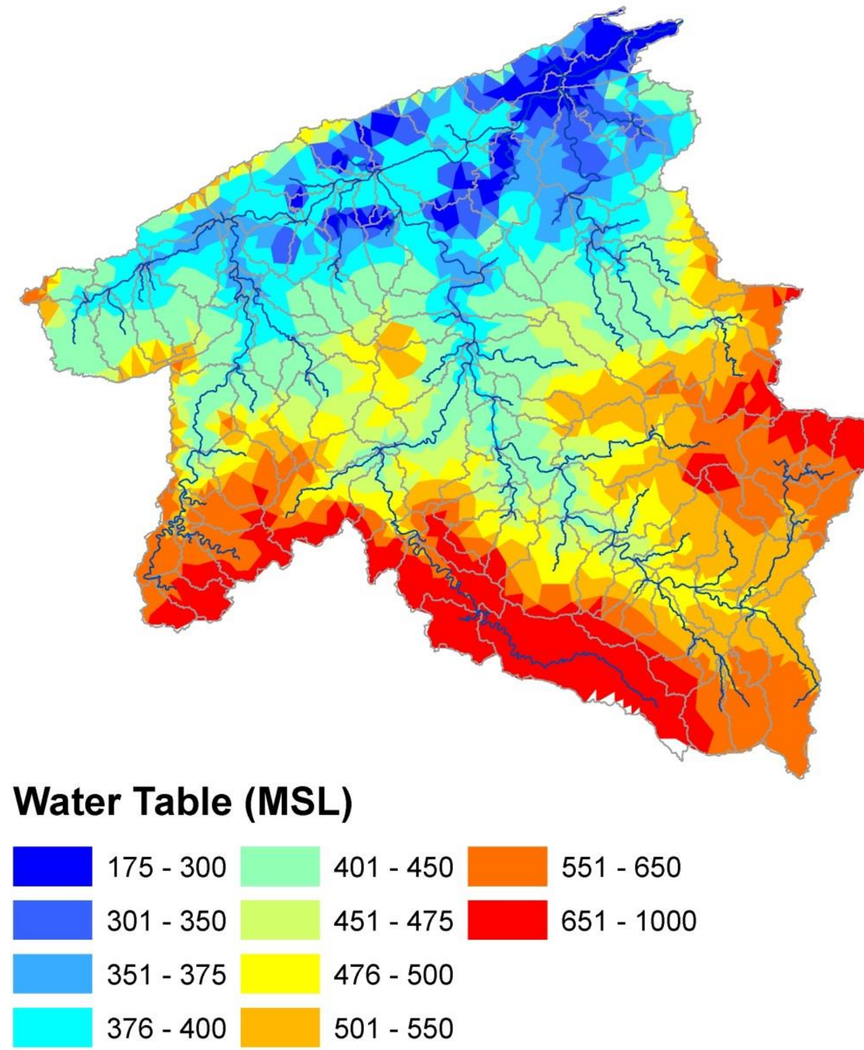
25.01 - 100.00


May 1977
(No ground water pumping)
Water Table (MSL)
175 - 300
301 - 350
351 - 375
376 - 400
401 - 450
451 - 475
476 - 500
501 - 550
551 - 650
651 - 1000
May 1977
(With water pumping)
Water Table (MSL)
175 - 300
301 - 350
351 - 375
376 - 400
401 - 450
451 - 475
476 - 500
501 - 550
551 - 650
651 - 1000

September 1977
(No ground water pumping)



September 1977
(With water pumping)



Assumptions

- The flow is horizontal
  - Vertical hydraulic gradient is assumed to be zero – Dupuit's assumptions
- The aquifer formation is primarily horizontal
- The aquifer can be anisotropic and heterogeneous in X and Y direction

Summary

- For simple cases, FEM formulation checked with analytical solution
- In future:
  - Application to a subbasin with higher base flow
    - Tifton watershed, Georgia
    - Upper North-Platte river, Nebraska