A Simplified Channel Routing Scheme Suitable for Adoption in SWAT Model

Dr. M. Perumal

Professor & Head

Department of Hydrology

Indian Institute of Technology Roorkee

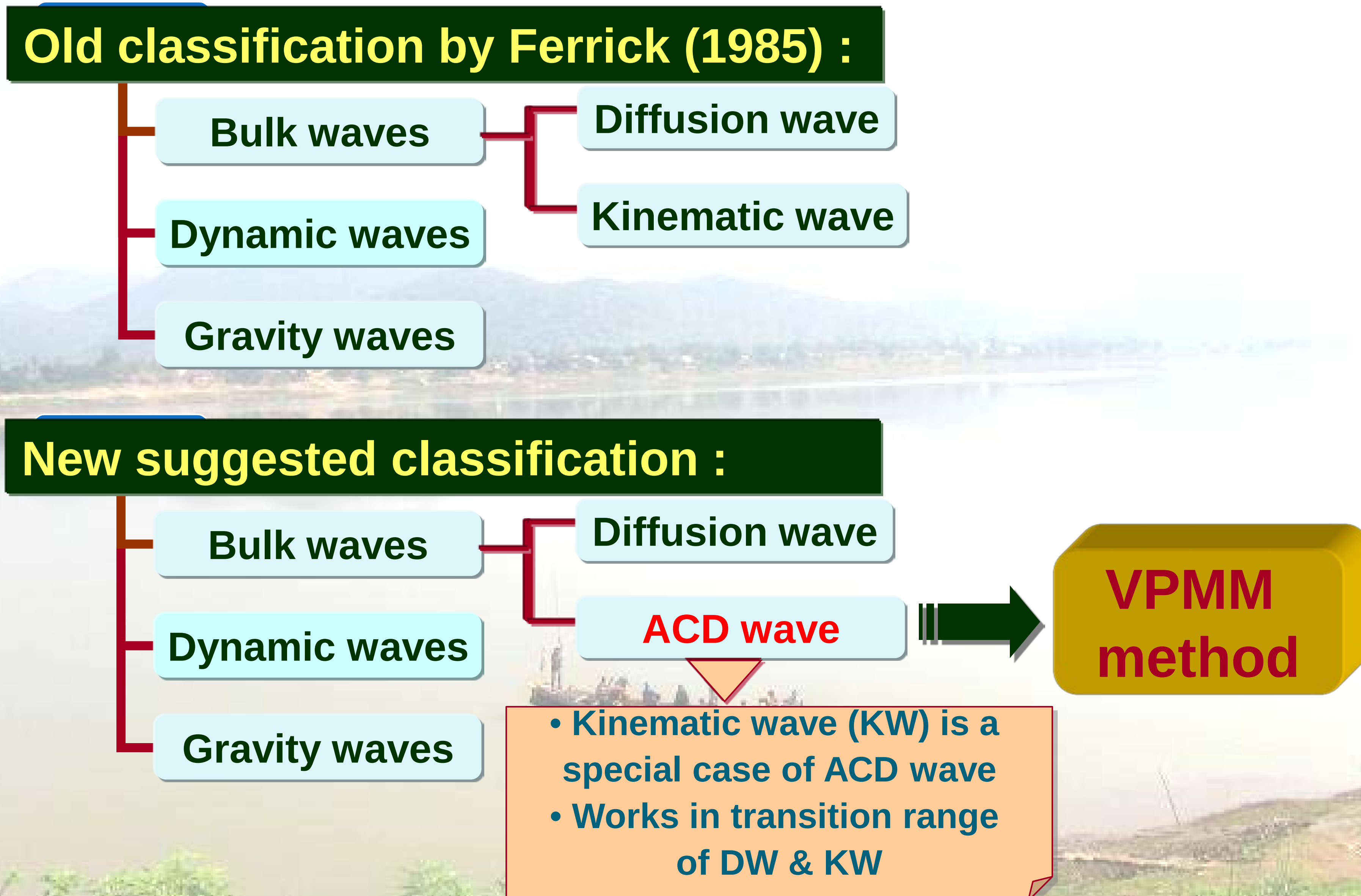
INDIA

Co-authors: Dr. B. Sahoo & Dr. C.M. Rao

Presentation Outline

- Linearity of channel routing scheme in SWAT model using classical Muskingum (CM) method
- Frameworks of variable parameter Muskingum-Cunge (VPMC) & Muskingum-Cunge-Todini (MCT) routing methods available in literature
- Development of variable parameter McCarthy-Muskingum (VPMM) routing method
- Performance evaluation criteria
- Numerical Application
    - Trapezoidal, Rectangular & Triangular channel sections
- Performance of CM, VPMC, MCT & VPMM methods
- Conclusions

Classification of River Flood Waves



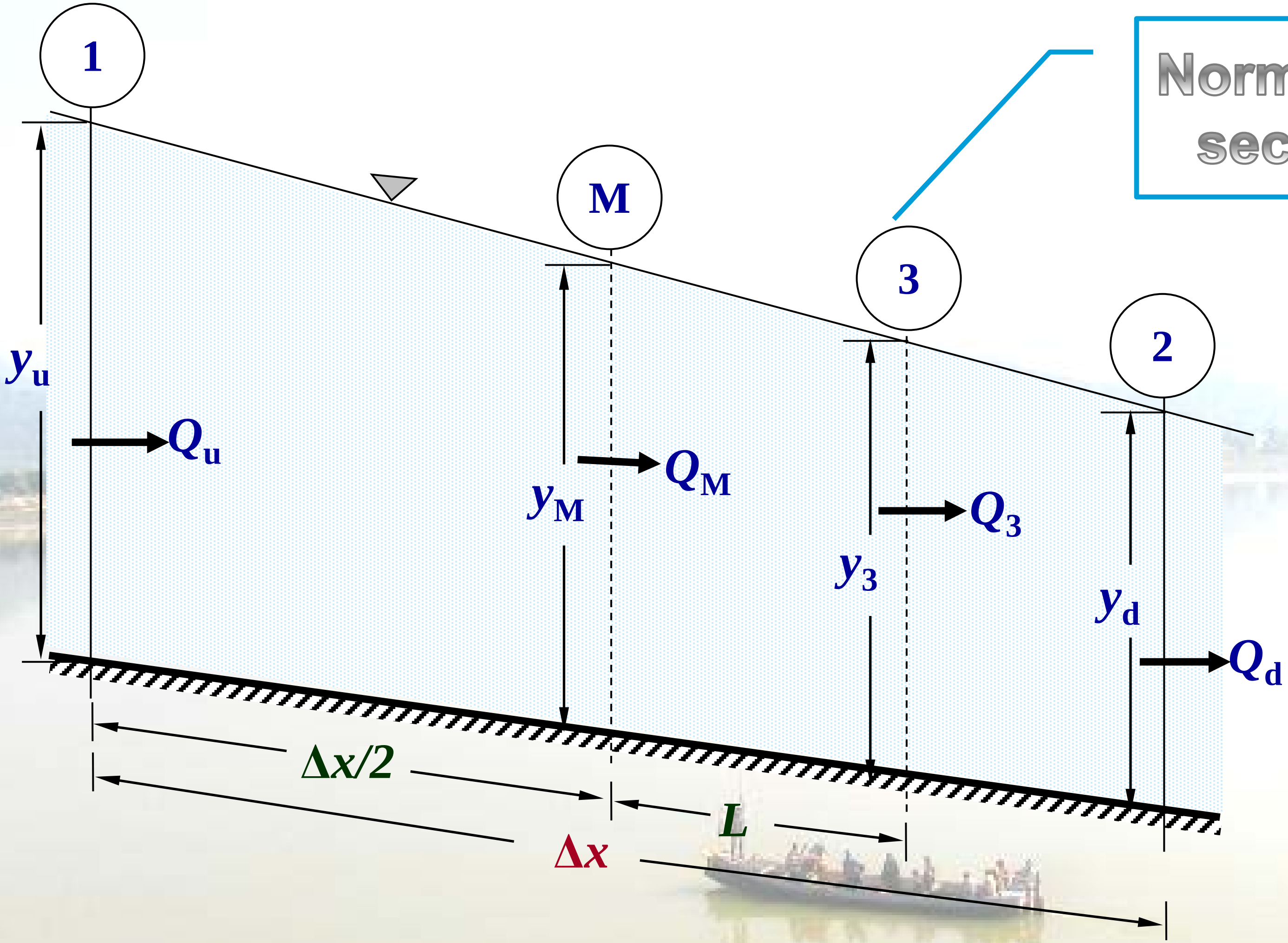
Development of VPMM Method

- VPMM- directly derived from Saint-Venant Equations.
- Can be used for routing flood waves in semi-infinite rigid bed prismatic channels having any cross-sectional shape.
- Allows simultaneous computations of stage and discharge hydrographs at any ungauged river site.

Hypothesis used:

- During steady flow in the channel reach, there exists one-to-one discharge - depth relationship; which is not valid in case of unsteady flows.
- During unsteady flow in the channel reach, the discharge observed at any section has its corresponding normal depth at the upstream section.

Definition Sketch of VPMM Method



Normal discharge section: $Q_3=Q_o$

$$L = \frac{Q_3}{2 S_o B_M c_{Mo}}$$

Routing parameters

$$K = \frac{\Delta x}{V_{Mo}}$$

$$\theta = 0.5 - L/\Delta x$$

Dr. M. Perumal, Professor & Head, Dept. of Hydrology, I.I.T. Roorkee, India

Framework of VPMM Method (Perumal & Ponce, 2012)

Routing equation:

$$O_{j+1} = \frac{\Delta t - 2.K_{j+1}.\theta_{j+1}}{\Delta t + 2.K_{j+1}.\left(1-\theta_{j+1}\right)}.I_{j+1} + \frac{\Delta t + 2.K_{j}.\theta_{j}}{\Delta t + 2.K_{j+1}.\left(1-\theta_{j+1}\right)}.I_{j} + \frac{-\Delta t + 2.K_{j}.\left(1-\theta_{j}\right)}{\Delta t + 2.K_{j+1}.\left(1-\theta_{j+1}\right)}.O_{j}$$

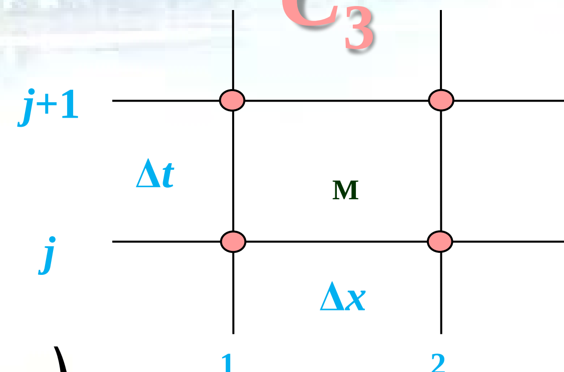
C_1 C_2 C_3

where:

$$K_{j+1} = \Delta x / V_{Mo,j+1}$$

$$\theta_{j+1} = 0.5 - Q_{3,j+1} / \left(2 S_o B_{Mo,j+1}\, c_{Mo,j+1}\, \Delta x\right)$$

$$Q_{3,j+1} = \theta_{j+1} I_{j+1} + (1-\theta_{j+1}) Q_{j+1}$$



Numerical scheme of VPMM method

For derivation details of VPMM method, click here

Framework of VPMC Method (*Ponce and Chaganti*, 1994)

Routing equation:

$$O_{j+1} = C_1 O_j + C_2 I_j + C_3 I_{j+1}$$

where:

$$C_1 = \frac{-K\theta + 0.5\Delta t}{K(1-\theta) + 0.5\Delta t} \quad C_2 = \frac{K\theta + 0.5\Delta t}{K(1-\theta) + 0.5\Delta t} \quad C_3 = \frac{K(1-\theta) - 0.5\Delta t}{K(1-\theta) + 0.5\Delta t}$$

$$K = \Delta x / c \qquad \theta = 0.5 - Q/(2 S_o B c \Delta x)$$

In CM method, the routing parameters remain constant at every routing time step so that $C_1+C_2+C_3=1$, making it mass conservative.

Framework of MCT Method (Todini, 2007)

Routing equation:

$$O_{j+1} = \frac{-1 + C^*_{j+1} + D^*_{j+1}}{1 + C^*_{j+1} + D^*_{j+1}} I_{j+1} + \frac{1 + C^*_j - D^*_j}{1 + C^*_{j+1} + D^*_{j+1}} \frac{C^*_{j+1}}{C^*_j} I_j + \frac{1 - C^*_j + D^*_j}{1 + C^*_{j+1} + D^*_{j+1}} \frac{C^*_{j+1}}{C^*_j} O_j$$

where:

$$C^*_j = (c_j / \beta_j)(\Delta t / \Delta x)$$

$$\beta_j = c_j / v_j$$

$$C^*_{j+1} = (c_{j+1} / \beta_{j+1})(\Delta t / \Delta x)$$

$$\beta_{j+1} = c_{j+1} / v_{j+1}$$

$$D^*_j = Q_j / (\beta_j B S_o c_j \Delta x)$$

$$D^*_{j+1} = Q_{j+1} / (\beta_{j+1} B S_o c_{j+1} \Delta x)$$

Performance Evaluation

- **Comparative evaluation of CM, VPMC, MCT & VPMM methods with respect to benchmark solutions of the Saint-Venant equations.**
- **Evaluated these methods by routing a given hypothetical flood wave in uniform prismatic Trapezoidal, Rectangular and Triangular cross-section channel reaches of different configurations.**

Performance Evaluation Criteria

$$\eta_q = \left[1 - \sum_{i=1}^{N} \left(Q_{oi} - Q_{ci} \right)^2 \Big/ \sum_{i=1}^{N} \left(Q_{oi} - \overline{Q}_{oi} \right)^2 \right] \times 100$$

Nash and Sutcliffe (1970)

$$EVOL = \left[\sum_{i=1}^{N} Q_{ci} \Big/ \sum_{i=1}^{N} I_i - 1 \right] \times 100$$

$$q_{per} = \left(q_{pc} / q_{po} - 1 \right) \times 100$$

$$t_{qper} = \left(t_{qpc} / t_{qpo} - 1 \right) \times 100$$

***Negative indicates– underestimation;**

***Positive indicates– over estimation**

Numerical Application

Parameters	Values
Gamma, γ	1.05, 1.15, 1.25, 1.50
Channel bed slope, S_o	0.002, 0.001, 0.0008, 0.0005, 0.0004, 0.0002, 0.0001
Manning's roughness, n	0.01, 0.02, 0.03, 0.04, 0.05
Initial discharge, Q_b (m^3/s)	100.0
Peak stage, y_p (m)	5.0, 8.0, 10.0, 12.0, 15.0
Time-to-peak stage, t_p (h)	5.0, 10.0, 15.0, 20.0
Channel bottom width, b (m)	100.0
Channel side slope, z	0.0, 1.0, 3.0, 5.0

 # Total reach length= 40 km
 # Δx = 1 km; Δx= 5 min.
 # Total no. of runs = 9847 (for each of the Saint-Venant, VPMC & VPMM methods)

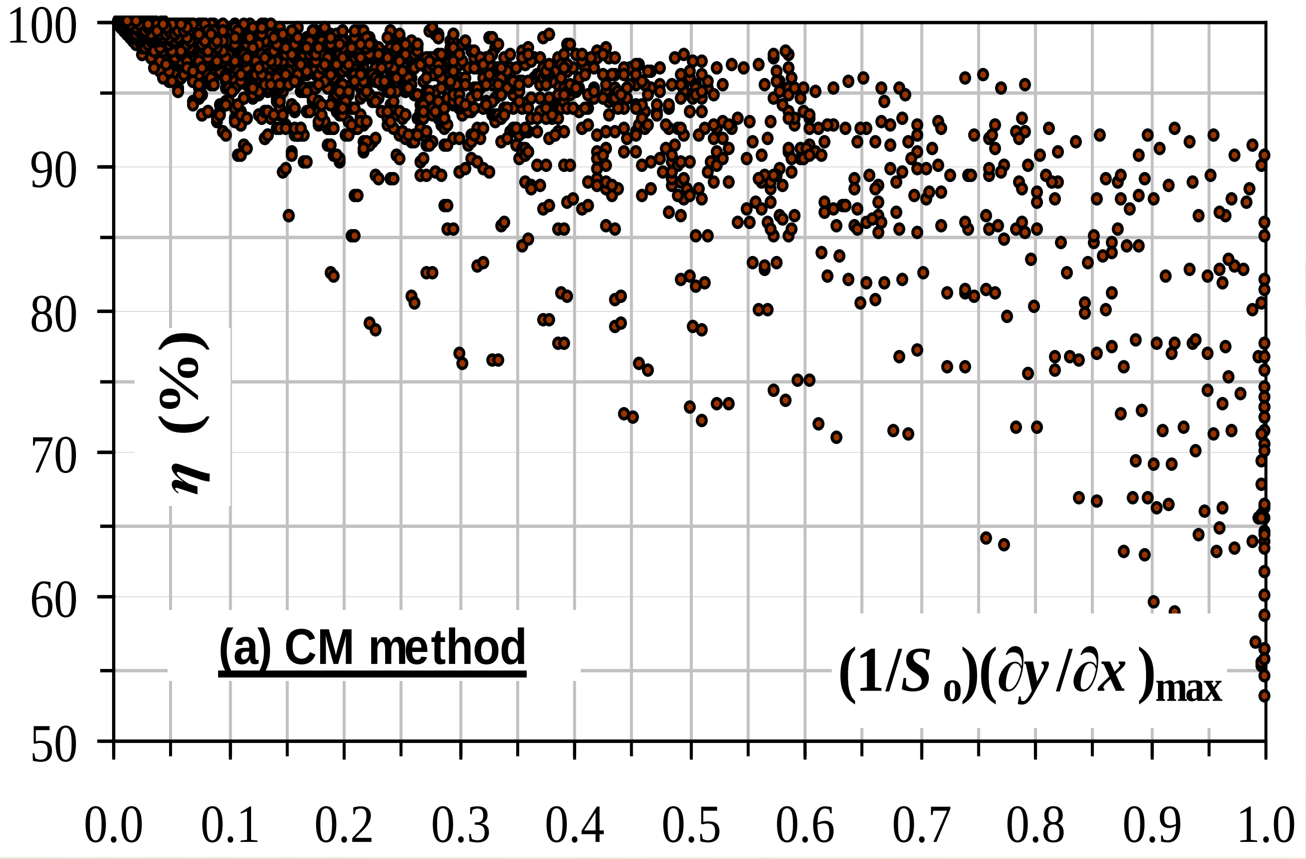
Upstream boundary condition:

$$y(0,t) = y_b + \left(y_p(t) - y_b\right)\left[\frac{t}{t_p}\right]^{1/(\gamma-1)} \exp\left[\frac{1-t/t_p}{\gamma-1}\right]$$

Inflow discharge hydrographs corresponding to the input stage hydrographs in the form of Pearson type III distribution are used to generate random sizes of discharge hydrographs.

Evaluation of CM, VPMC, MCT & VPMM Methods

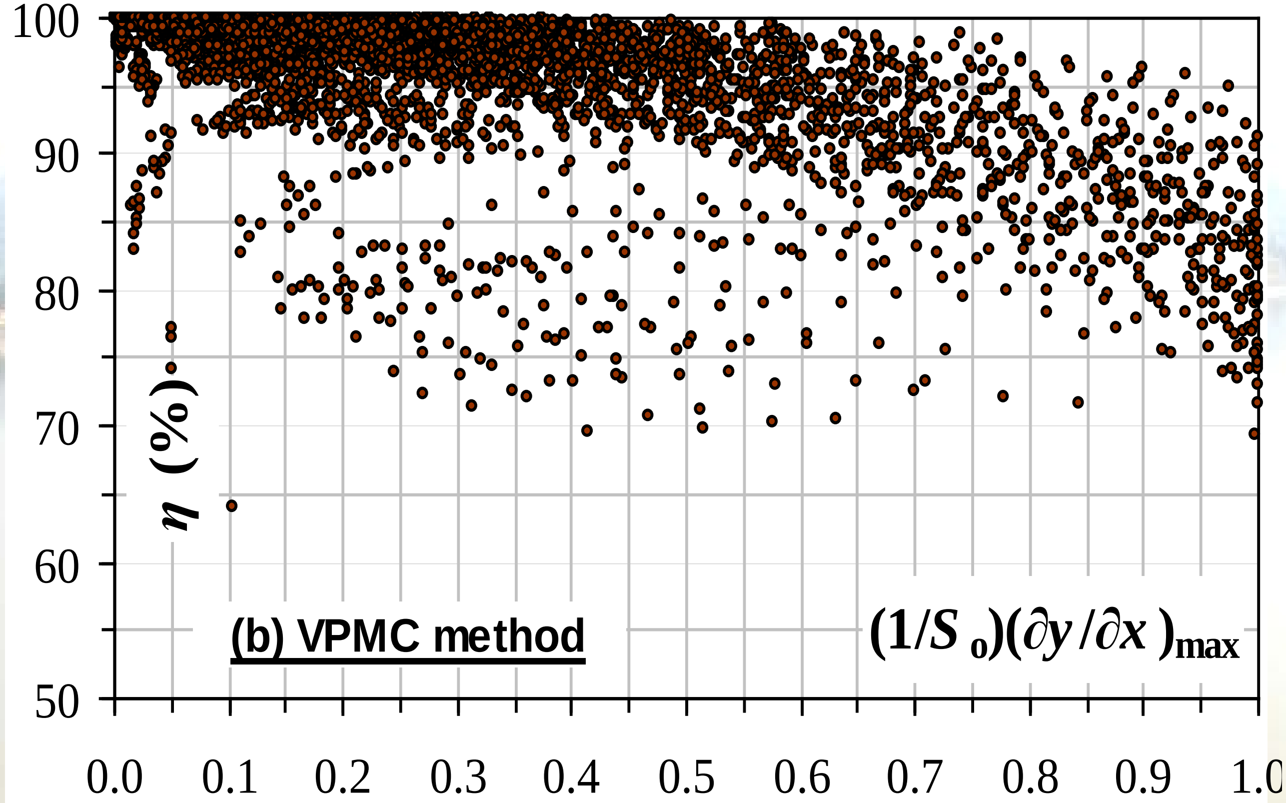
Nash-Sutcliffe Efficiency



CM always performs with
η≥50% for all the cases except five runs
η≥95% for all the runs except 115 (1.17%) cases

Evaluation of CM, VPMC, MCT & VPMM Methods

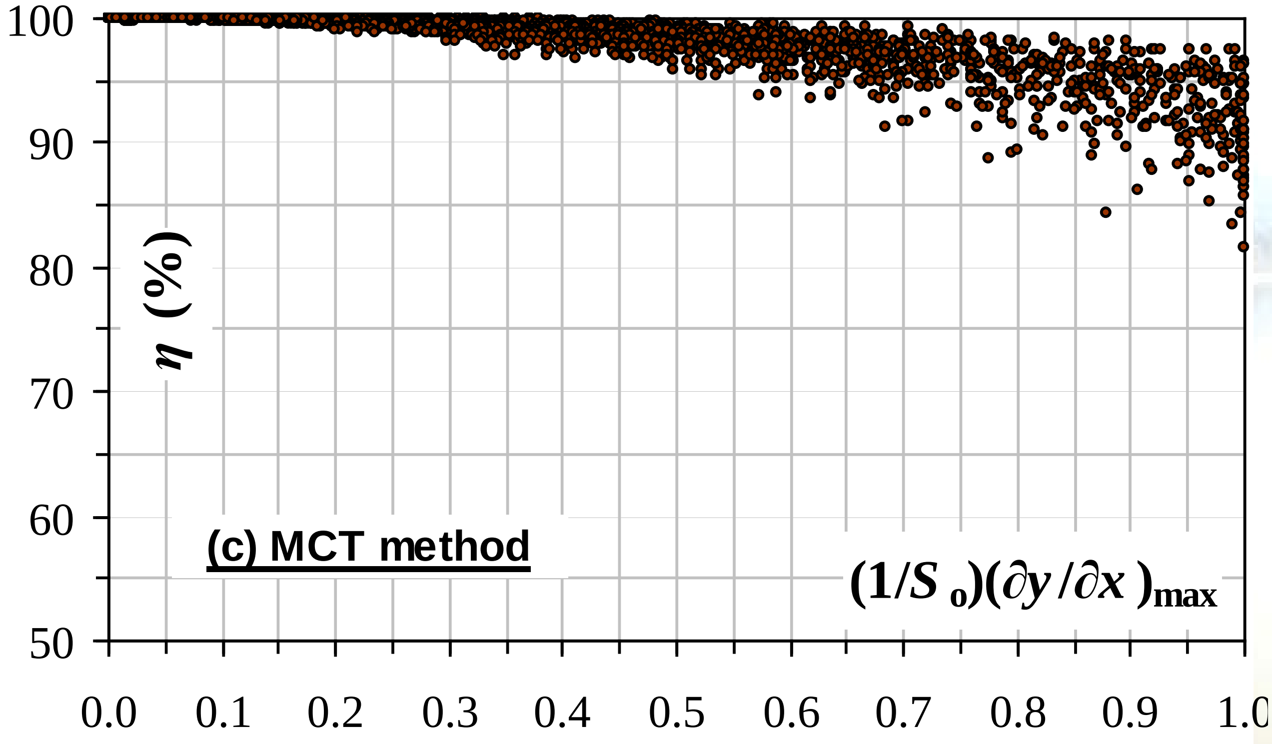
Nash-Sutcliffe Efficiency



CM always performs with
η≥90% for all the cases except five runs
η≥95% for all the runs except 115 (1.17%) cases

Evaluation of CM, VPMC, MCT & VPMM Methods

Nash-Sutcliffe Efficiency

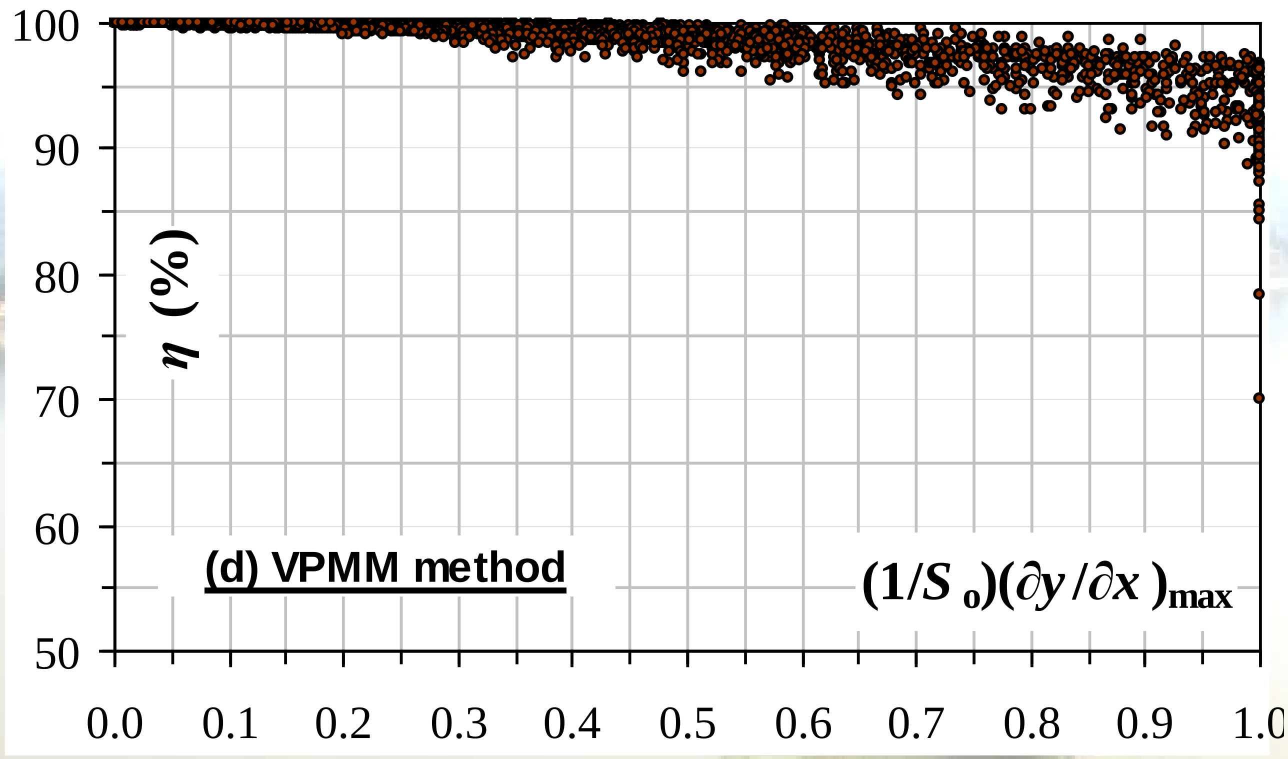


MCT always performs with

η≥80% for all the cases except five runs

Evaluation of CM, VPMC, MCT & VPMM Methods

Nash-Sutcliffe Efficiency

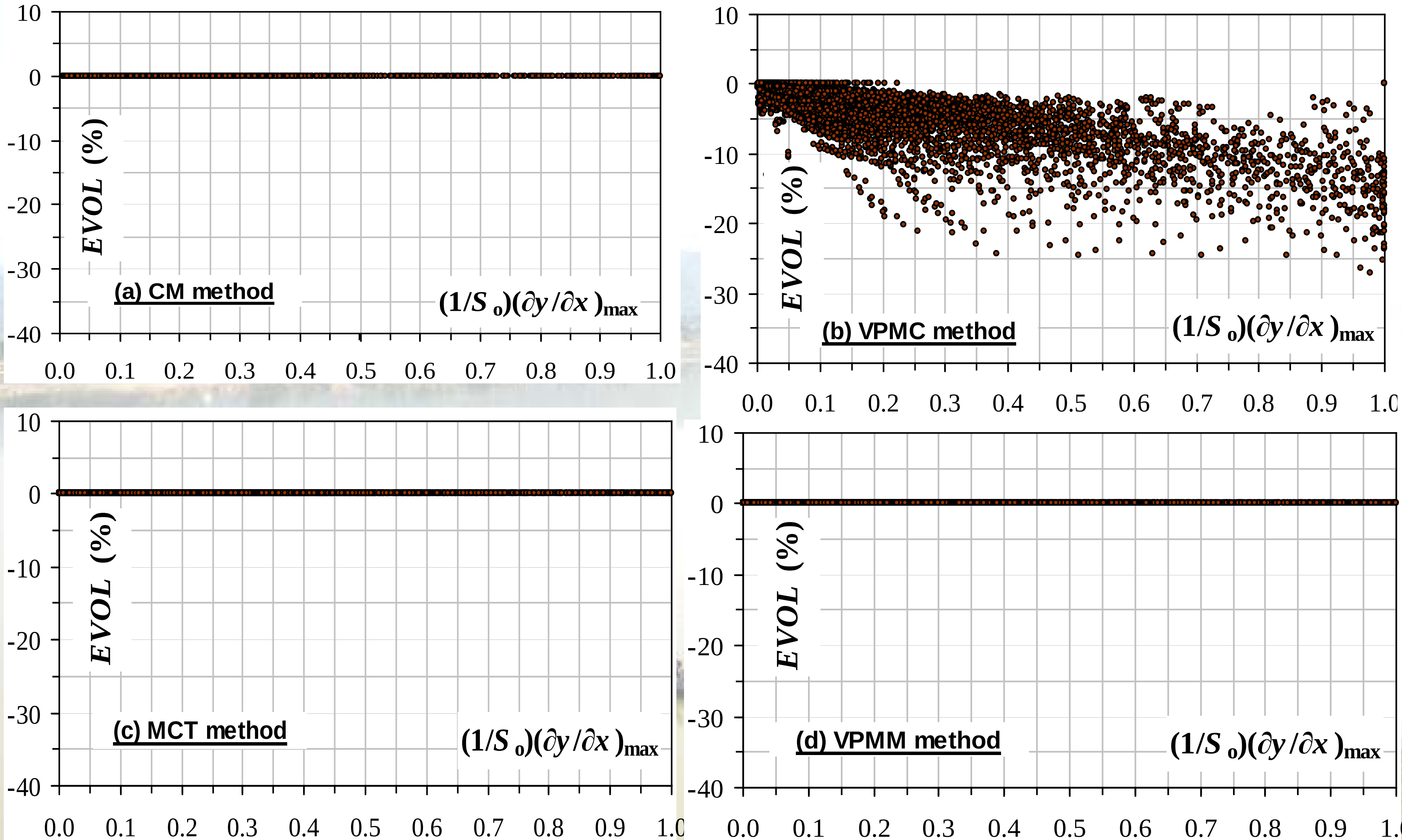


VPMM always performs with

η≥90% for all the cases except nine runs

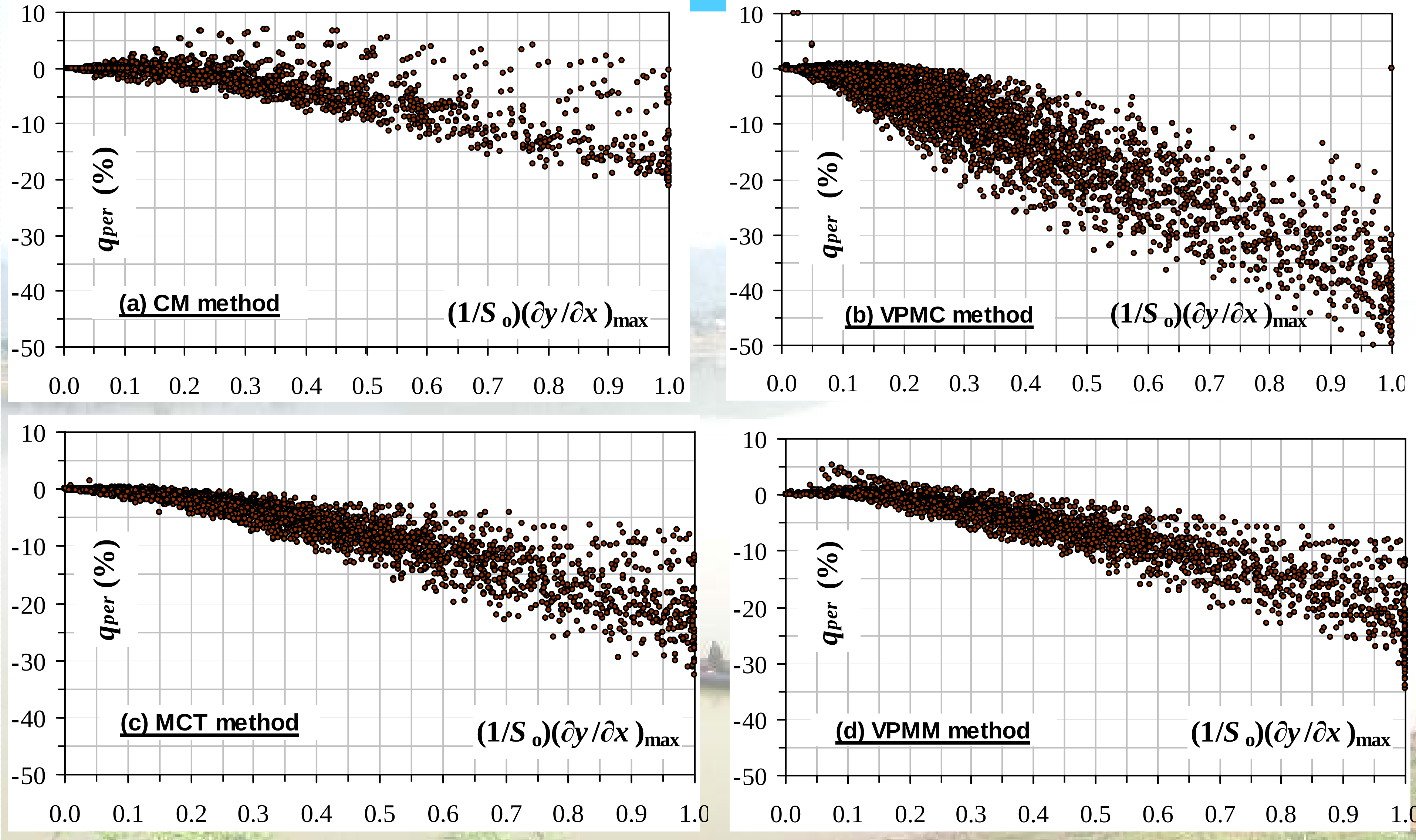
Evaluation of CM, VPMC, MCT & VPMM Methods

Error in Volume



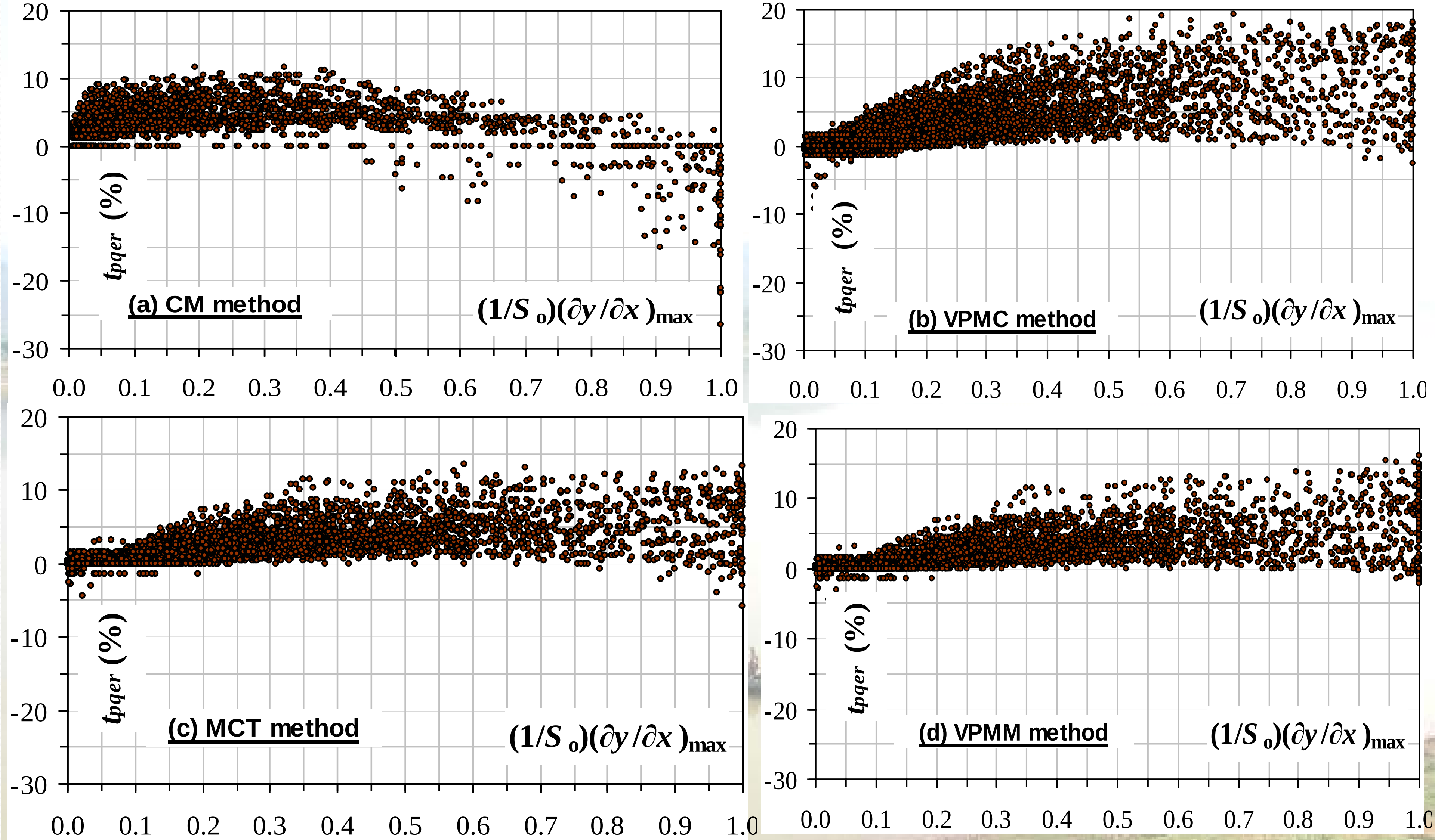
Evaluation of CM, VPMC, MCT & VPMM Methods

Error in Peak Discharge



Evaluation of CM, VPMC, MCT & VPMM Methods

Error in Time-to-peak Discharge



Conclusions

- VPMM method consistently performs better than the MC & VPMC method.

- VPMM, MCT & MC methods are fully volume conservative, whereas the VPMC method has the tendency to always loose mass.

- Due to the nonlinearity dynamics of river flood convection, the MC method should not be used in SWAT model.

- Since the VPMM method has a sound physical basis, it may be preferred over the other existing methods for its incorporation in the SWAT model for dealing with various field problems.

Thank You

Mathematical Formulation of VPMM Method

Saint-Venant Equations:

Continuity equation: $$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = 0 \quad (1)$$

Momentum eqn. $$S_f = S_0 - \frac{\partial y}{\partial x} - \frac{v}{g}\frac{\partial v}{\partial x} - \frac{1}{g}\frac{\partial v}{\partial t} \quad (2)$$

S_o = bed slope; S_f = friction slope; $\partial y/\partial x$ = water surface slope; $(v/g)(\partial v/\partial x)$ = convective acceleration; $(1/g)(\partial v/\partial t)$ = local acceleration.

Magnitudes of various terms in eqn. (2) are usually small in comparison with S_o [*Henderson,* 1966*; NERC,* 1975].

Important steps of derivation:

- **Unsteady flow relationship:** $$Q\left(y, \partial y/\partial x\right) = Q(y)\sqrt{S_f / S_0}$$ **(3)**

- **Considering ∂y/∂x≠0 & ∂²y/∂x²→0:** $$\frac{\partial Q}{\partial x} = \frac{\partial (Av)}{\partial x} = Bc\frac{\partial y}{\partial x}$$ **(4)**

- **Celerity of the flood wave (Henderson,1966; NERC,1975):** $$c = \frac{dQ}{dA} = \left[1 + \frac{2}{3}\frac{PdR/dy}{dA/dy}\right]v$$ **(5)**

- **Velocity by the Manning's friction law:** $$v = \frac{1}{n}R^{2/3}S_f^{1/2}$$ **(6)**

- **Using equations (1), (2) (4), (5), & (6) friction slope:** $$S_f = S_o\left\{1 - \frac{1}{S_o}\frac{\partial y}{\partial x}\left[1 - \frac{4}{9}F^2\left(\frac{PdR/dy}{dA/dy}\right)^2\right]\right\}$$ **(7)**

- **Where Froude number:** $$F = \left(\frac{v^2\, dA/dy}{gA}\right)^{1/2}$$ **(8)**

Back to Slide 6

To Eq. (14)

- Using Eq. (7) in (3), unsteady discharge:

$$Q = Q\left(y, \frac{\partial y}{\partial x}\right) = Q_o(y)\left\{1 - \frac{1}{S_0}\frac{\partial y}{\partial x}\left[1 - \frac{4F^2}{9}\left(\frac{P}{B}\frac{dR}{dy}\right)^2\right]\right\}^{\frac{1}{2}} \quad (9)$$

- Similarly, unsteady velocity:

$$v = v\left(y, \frac{\partial y}{\partial x}\right) = v_o(y)\left\{1 - \frac{1}{S_0}\frac{\partial y}{\partial x}\left[1 - \frac{4F^2}{9}\left(\frac{P}{B}\frac{dR}{dy}\right)^2\right]\right\}^{\frac{1}{2}} \quad (10)$$

- Hydraulic continuity Eq. (1):

$$\frac{\partial}{\partial t}\left(\frac{Q}{v}\right) + \left(\frac{\partial Q}{\partial x}\right) = 0 \quad (11)$$

- Using Eqs. (9) & (10) in (11):

$$\frac{\partial}{\partial t}\left(\frac{Q_0}{v_0}\right) + \frac{\partial Q}{\partial x} = 0 \quad (12)$$

- Normal discharge in Eq. (9) can be rewritten as:

$$Q_o = Q_o(y) = Q\left\{1 - \frac{1}{S_0}\frac{\partial y}{\partial x}\left[1 - \frac{4F^2}{9}\left(\frac{P}{B}\frac{dR}{dy}\right)^2\right]\right\}^{-\frac{1}{2}} \quad (13)$$

Back to Slide 6

•Using binomial series expansion in Eq. (13) with $(1/S_0)(\partial y/\partial x) \ll 1$ & Eq. (4):

$$Q_0 = Q + \frac{Q}{2S_0Bc}\left[1 - \frac{4F^2}{9}\left(\frac{P}{B}\frac{dR}{dy}\right)^2\right]\frac{\partial Q}{\partial x} \quad (14)$$

• Which is in the form of:

$$Q_0 = Q + \frac{a}{c}\frac{\partial Q}{\partial x} \quad (15)$$

Where:

$$a = \frac{Q}{2S_0B}\left[1 - \frac{4F^2}{9}\left(\frac{P}{B}\frac{dR}{dy}\right)^2\right] \quad (16)$$

• Celerity can also be written as:

$$c = c\left(y, \frac{\partial y}{\partial x}\right) = c_o(y)\left\{1 - \frac{1}{S_0}\frac{\partial y}{\partial x}\left[1 - \frac{4F^2}{9}\left(\frac{P}{B}\frac{dR}{dy}\right)^2\right]\right\}^{\frac{1}{2}} \quad (17)$$

• Using Eqs. (9) & (17) in (15):

$$Q_0 = Q + \frac{a_0}{c_0}\frac{\partial Q}{\partial x} \quad (18)$$

Where:

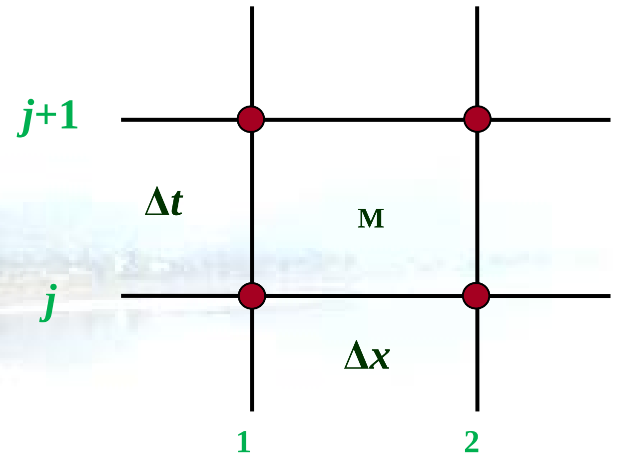
$$a_0 = \frac{Q_0}{2S_0B}\left[1 - \frac{4F^2}{9}\left(\frac{P}{B}\frac{dR}{dy}\right)^2\right] \quad (19)$$

Back to Slide 6

- Using Eq. (18) in continuity Eq. (12):

$$\frac{\partial}{\partial t}\left[\frac{Q+\dfrac{a_0}{c_0}\dfrac{\partial Q}{\partial x}}{v_0}\right]+\frac{\partial Q}{\partial x}=0 \qquad (19)$$

- Expressing Eq. (19) at the centre point 'M' of the finite difference grid:



Finite-difference grid representation of the VPMM method computational scheme

$$\frac{1}{\Delta t}\left[\frac{Q_M^{j+1}+\dfrac{a_0}{c_0}\dfrac{\partial Q}{\partial x}\Big|_M^{j+1}}{v_{0,M}^{j+1}}-\frac{Q_M^{j}+\dfrac{a_0}{c_0}\dfrac{\partial Q}{\partial x}\Big|_M^{j}}{v_{0,M}^{j}}\right]+\frac{1}{2}\left[\frac{\partial Q}{\partial x}\Big|_M^{j+1}+\frac{\partial Q}{\partial x}\Big|_M^{j}\right]=0 \qquad (20)$$

Back to Slide 6

- Eq. (20) can be reorganized as:

$$\frac{\Delta x}{v_{0,M}^{j+1}}\left[\left(\frac{1}{2}-\frac{a_0}{c_0}\bigg|_M^{j+1}\frac{1}{\Delta x}\right)Q_i^{j+1}+\left(\frac{1}{2}+\frac{a_0}{c_0}\bigg|_M^{j+1}\frac{1}{\Delta x}\right)Q_{i+1}^{j+1}\right]$$

$$-\frac{\Delta x}{v_{0,M}^{j}}\left[\left(\frac{1}{2}-\frac{a_0}{c_0}\bigg|_M^{j}\frac{1}{\Delta x}\right)Q_i^{j}+\left(\frac{1}{2}+\frac{a_0}{c_0}\bigg|_M^{j}\frac{1}{\Delta x}\right)Q_{i+1}^{j}\right]$$

$$=\Delta t\left[\frac{Q_i^{j+1}+Q_i^j}{2}-\frac{Q_{i+1}^{j+1}+Q_{i+1}^j}{2}\right] \qquad (21)$$

- Which can be written as:

$$K^{j+1}\left[\theta^{j+1}Q_i^{j+1}+(1-\theta^{j+1})Q_{i+1}^{j+1}\right]-K^j\left[\theta^jQ_i^j+(1-\theta^j)Q_{i+1}^j\right]$$

$$=\left[\frac{Q_i^{j+1}+Q_i^j}{2}-\frac{Q_{i+1}^{j+1}+Q_{i+1}^j}{2}\right]\Delta t \qquad (22)$$

Back to Slide 6

- **Where:** $K^{j+1} = \dfrac{\Delta x}{v_{0,M}^{j+1}}$

$$\theta^{j+1} = \left(\frac{1}{2} - \frac{a_0}{c_0} \Bigg|_M^{j+1} \frac{1}{\Delta x} \right) = \frac{Q_{0,M}}{2 S_0 B_M c_{0,M} \Delta x} \left[1 - \frac{4}{9} F_M^2 \left(\frac{P}{B} \frac{dR}{dy} \right)_M^2 \right]^{j+1}$$

$$F_M = \left(\frac{Q_M^2 B_M}{g A_M^3} \right)^{\frac{1}{2}}$$

- Eq. (22) **can also be written in the form of the classical Muskingum routing equation as:**

$$Q_{i+1}^{j+1} = C_1 Q_i^{j+1} + C_2 Q_i^j + C_3 Q_{i+1}^j \qquad \textbf{(23)}$$

- **Where:**

$$C_1 = \frac{\Delta t - 2.K_{j+1}.\theta_{j+1}}{\Delta t + 2.K_{j+1}.\left(1 - \theta_{j+1}\right)} \quad C_2 = \frac{\Delta t + 2.K_j.\theta_j}{\Delta t + 2.K_{j+1}.\left(1 - \theta_{j+1}\right)} \quad C_3 = \frac{-\Delta t + 2.K_j.\left(1 - \theta_j\right)}{\Delta t + 2.K_{j+1}.\left(1 - \theta_{j+1}\right)} \qquad \textbf{(24)}$$

Back to Slide 6

- The reach storage at any computational time level can be given by:

$$S^{j+1} = K^{j+1}\left[Q_{i+1}^{j+1} + \theta^{j+1}(Q_i^{j+1} - Q_{i+1}^{j+1})\right]$$

- Where:

Prism storage = $K^{j+1} Q_{i+1}^{j+1}$

Wedge storage = $K^{j+1}\theta^{j+1}(Q_i^{j+1} - Q_{i+1}^{j+1})$

- Corresponding downstream stage hydrograph can be computed using Eq. (4) as:

$$y_{i+1}^{j+1} = y_M^{j+1} + \frac{(Q_{i+1}^{j+1} - Q_M^{j+1})}{B_M c_M^{j+1}}$$

This proves that the classical Muskingum method advocated by McCarthy (1938) has the physical basis

Back to Slide 6