Representation of modeling errors by AR(1) process and uncertainty analysis of SWAT model under Bayesian approach

Presented by

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Presentation outline

- Introduction
- Objectives
- Methodology- study area, data
- First order autoregressive [AR(1)] model and likelihood function
- Results and posterior checks
- Conclusions

Introduction

- Model parameters are estimated such that modeling results are close to observations.
- Differences between observed and simulated responses - modeling errors
- Uncertainty - input, parameters, structure and output.

Introduction

Uncertainty analysis (UA) methods

- ❑ **Type 1:** All uncertainties - represented by **parameter uncertainty** [SUFI-2, GLUE]
- ❑ **Type 2:** All uncertainties -considered **implicitly by introducing additive** error models to outputs.
- ❑ For SWAT- Type 1 is commonly used.

Introduction

Type 2 UA method :

- Autoregressive (AR) models- correlated errors.
- Data transformation – non-constant variance (heteroscedastic) and non-normality.

Objectives

- To represent the modeling errors by first order AR [AR(1)] model via the likelihood function.
- To evaluate the likelihood function for UA of SWAT model.
- To quantify parameter and simulation uncertainties.
- To check the adequacy of the likelihood function for representing errors.

Methodology

- SWAT model - calibration under Bayesian approach.
- Shuffled Complex Evolution Metropolis (SCEM-UA) algorithm -analyze the posterior probability distribution function (pdf).
- SCEM-UA - Markov Chain Monte Carlo (MCMC) sampler and a global optimization tool.
- Parameter uncertainty - estimated from posterior pdf
- Optimum parameters- at the maximum of posterior density.

Bayesian theory

- Posterior probability distribution of model parameters, $p(\theta|y)$ is expressed as:

$$p(\theta|y) \propto \underbrace{p(\theta)}_{\text{Prior distribution of parameters}}\underbrace{p(y|\theta)}_{\text{Likelihood function}} \qquad (1)$$

AR(1) model and likelihood function

AR(1) model: $e_t = \phi_1 e_{t-1} + v_t \quad v_t \approx N\left(0, \sigma_v^2\right)$

First order AR model parameter

Likelihood function:

$$l(\{\theta\}, \phi_1, \{Q\}_{obs}) = (2\pi)^{-\frac{n}{2}} \left(\frac{\sigma_v{}^2}{1-\phi_1{}^2} \right)^{-\frac{n}{2}} \exp\left[-\frac{1}{2}\sigma_v^{-2} \left\{ \left(1-\phi_1{}^2\right) e_1{}^2 + \left(\sum_{t=2}^{n} (v_t)^2 \right) \right\} \right]$$

Calibration parameters and their prior distributions

Curve Number: a__CN2.mgt

≈ U (-5.0, 5.0)

Available water holding capacity: a__SOL_AWC( ).sol

≈ U (-0.05, 0.05)

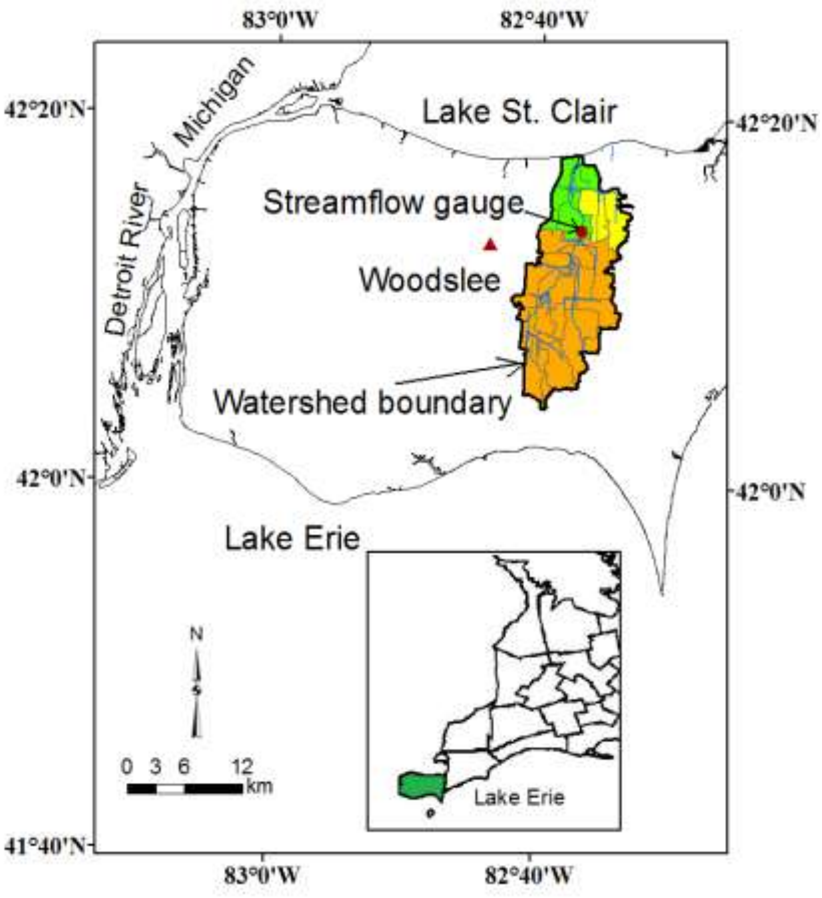
Plant uptake compensation factor: a__EPCO.bsn

≈ U (-0.05, 0.05)

Soil evaporation compensation factor: a__ESCO.bsn

≈ U (-0.05, 0.05)

Study area, data



Ruscom River watershed

Area : 175 km^2

Topography : Level to slightly undulating

Soil: Clayey with some sandy soils in southern part

Land use: Agriculture

An. av. prep (mm) : 920

Temp (C): -4.5^0 (Jan), 22.7^0 (Jul)

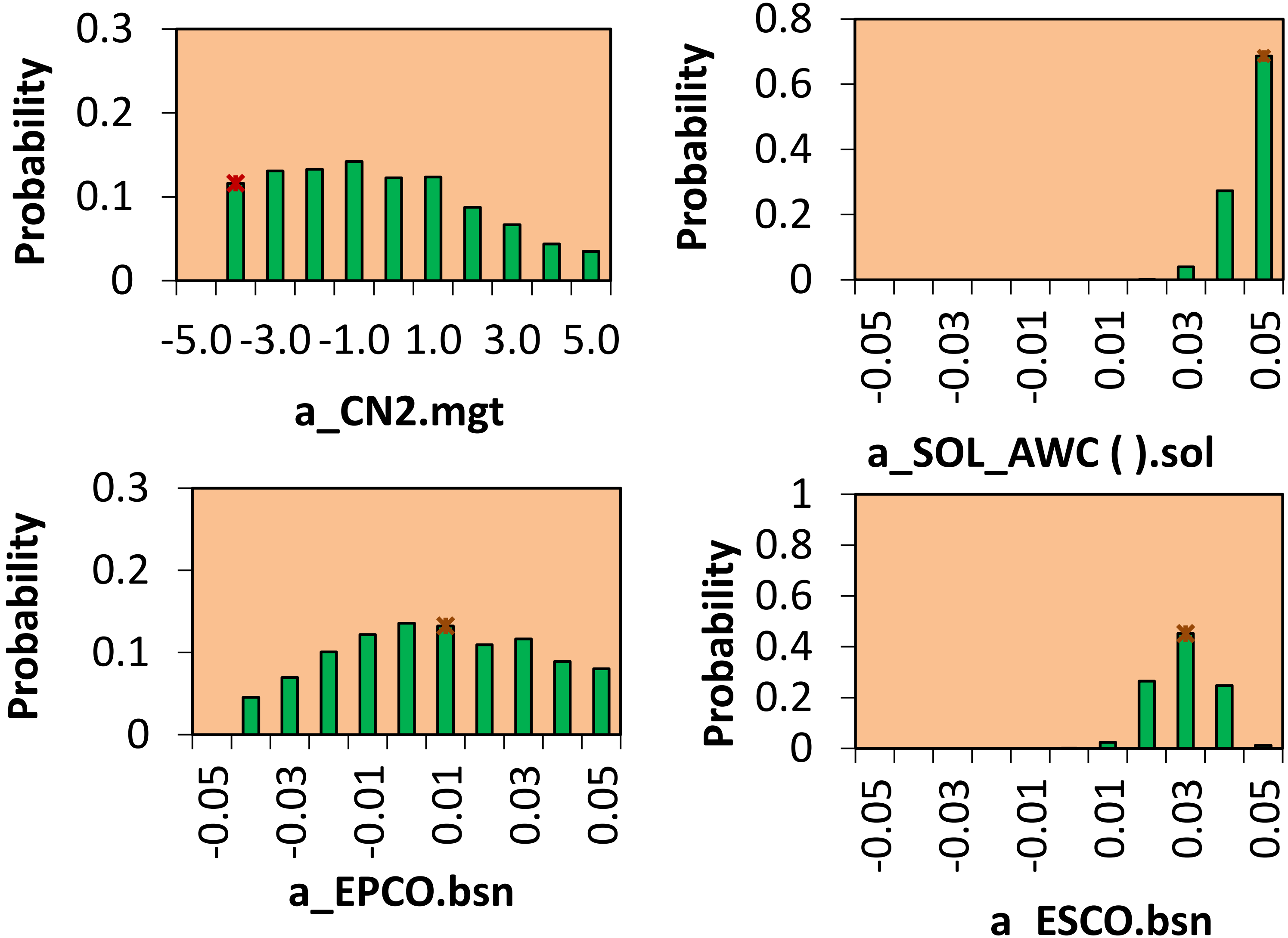
Calibration yrs: 1990-1993

Validation yrs: 1980-1983

Warm-up: 1 yr

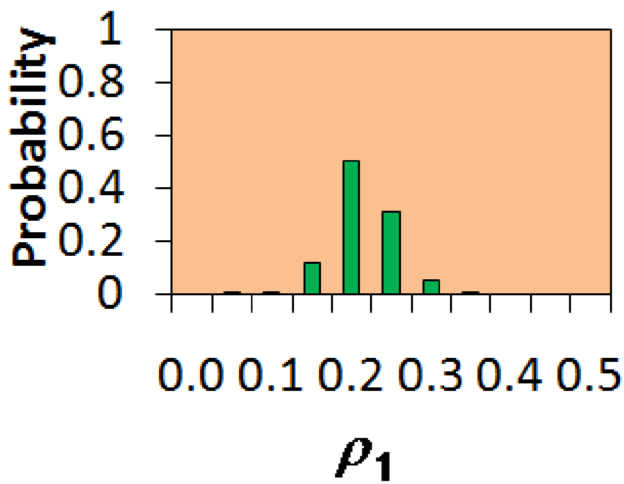
Watershed delineation: 31 sub-basins, 132 HRU

Marginal posterior pdf of SWAT model parameters



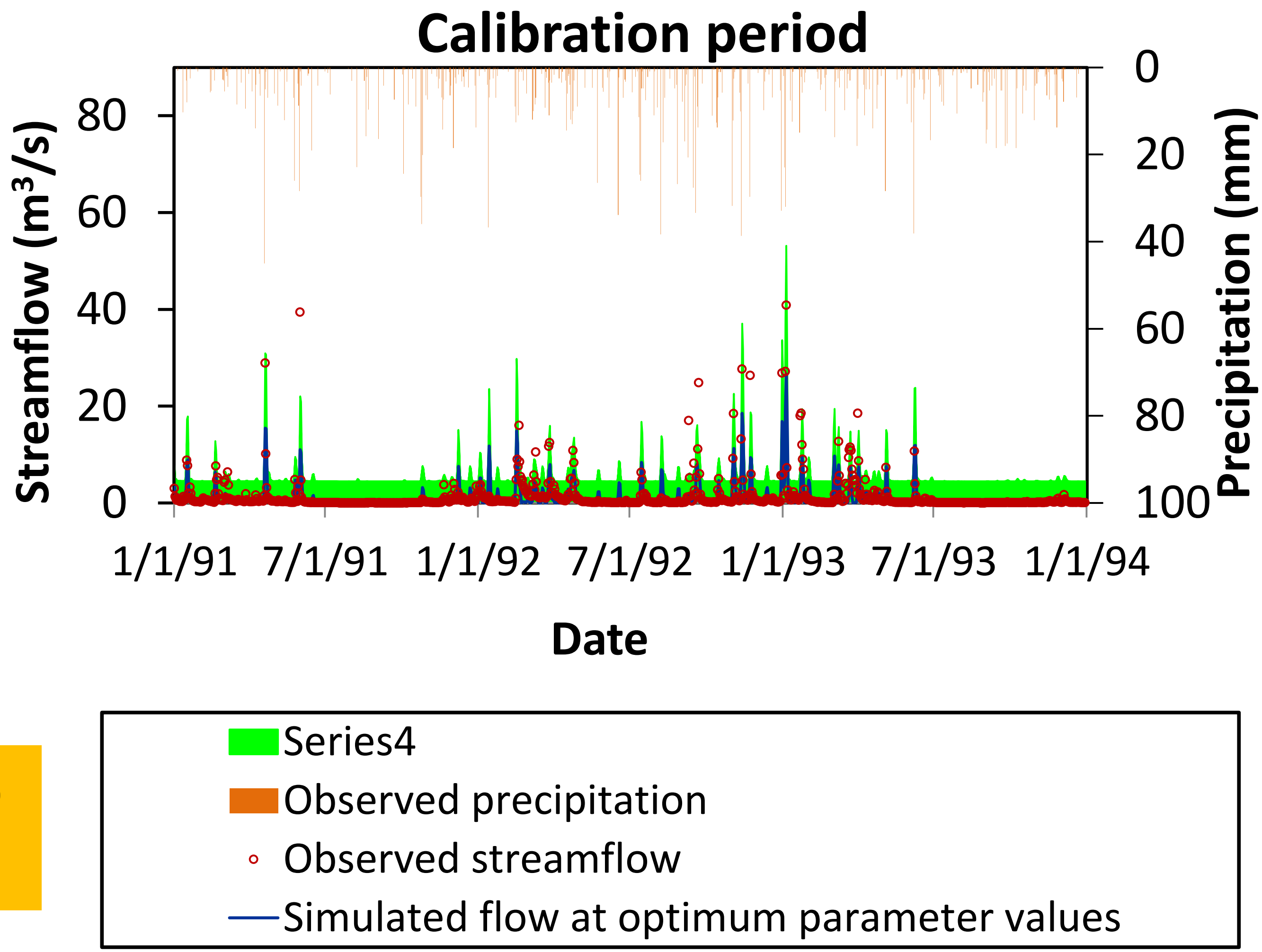
Parameter uncertainty

Parameter uncertainty estimation	
SWAT model parameters	95% confidence limits
a_CN2.mgt	(-5.81, 3.93)
a__SOL_AWC ( ).sol	(0.03, 0.05)
a__EPCO.bsn	(-0.05, 0.05)
a__ESCO.bsn	(0.01, 0.04)



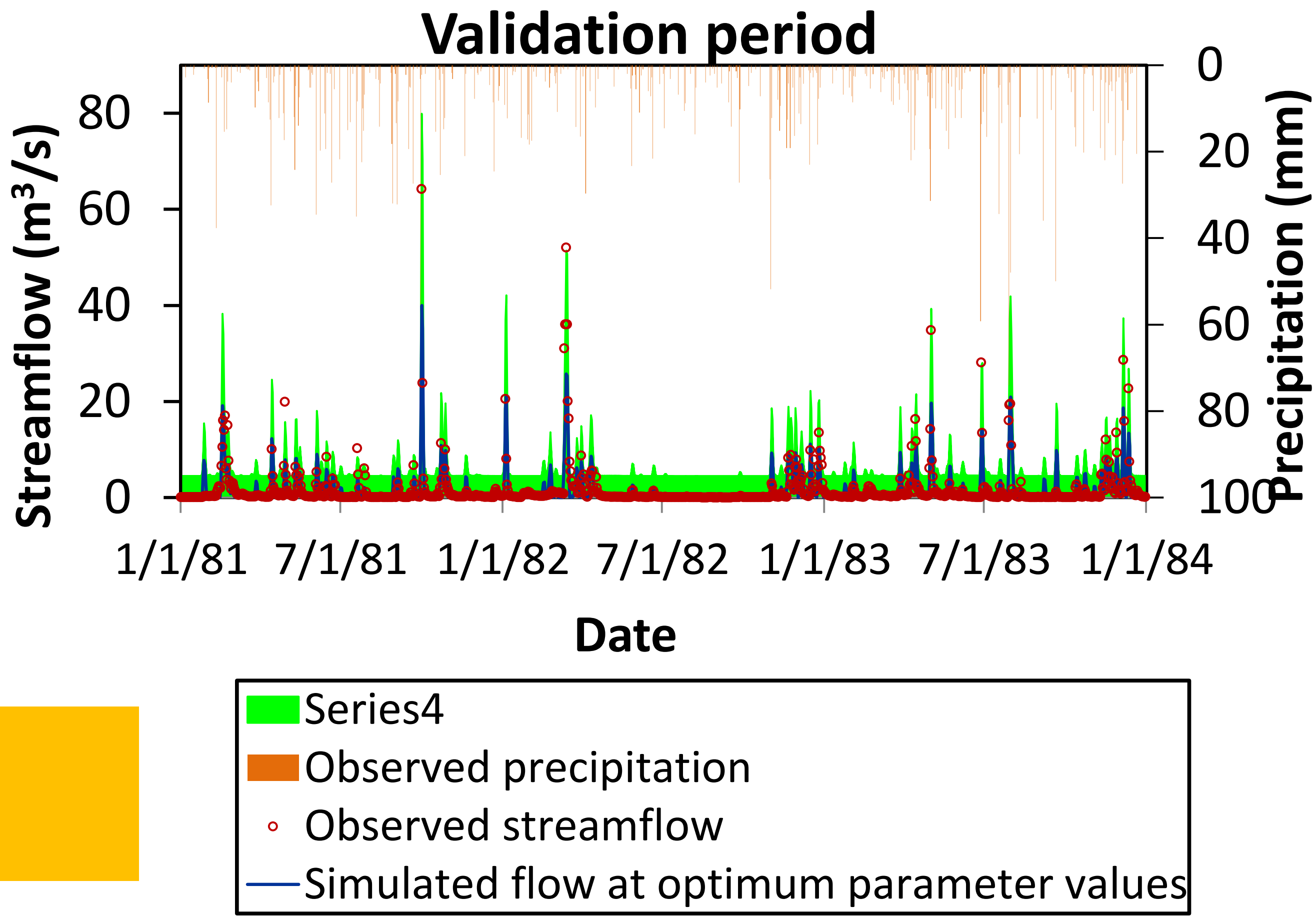
Marginal posterior pdf of AR(1) model parameter

Model simulation uncertainty: calibration



NS: 0.55 (daily)
0.90 (monthly)

Model simulation uncertainty: validation

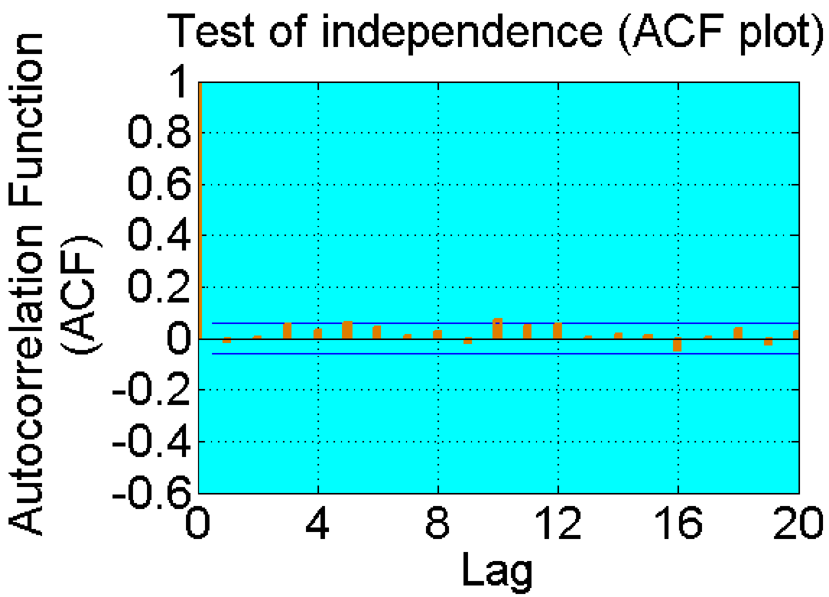


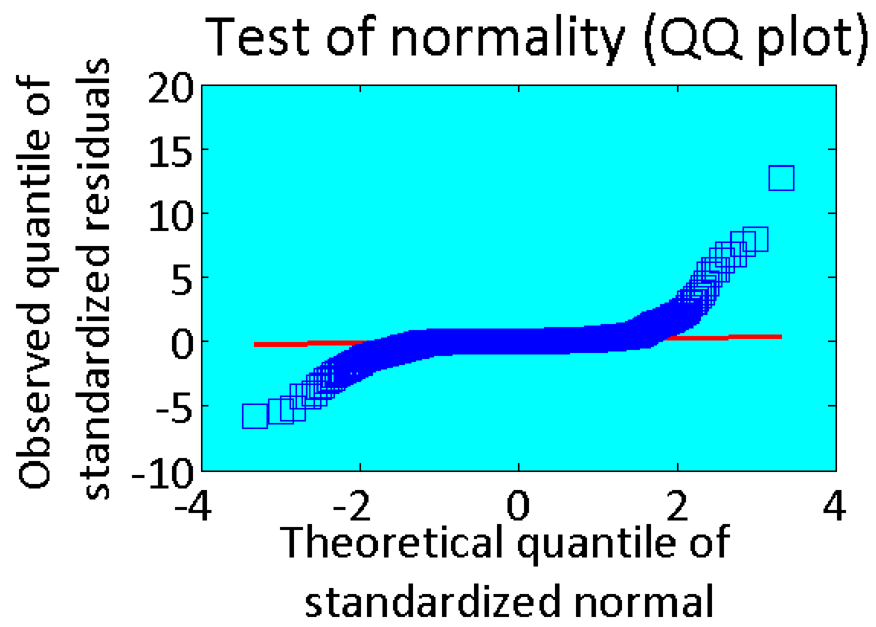
NS: 0.69 (daily)
0.80 (monthly)

Coverage of observed streamflow data by prediction uncertainty

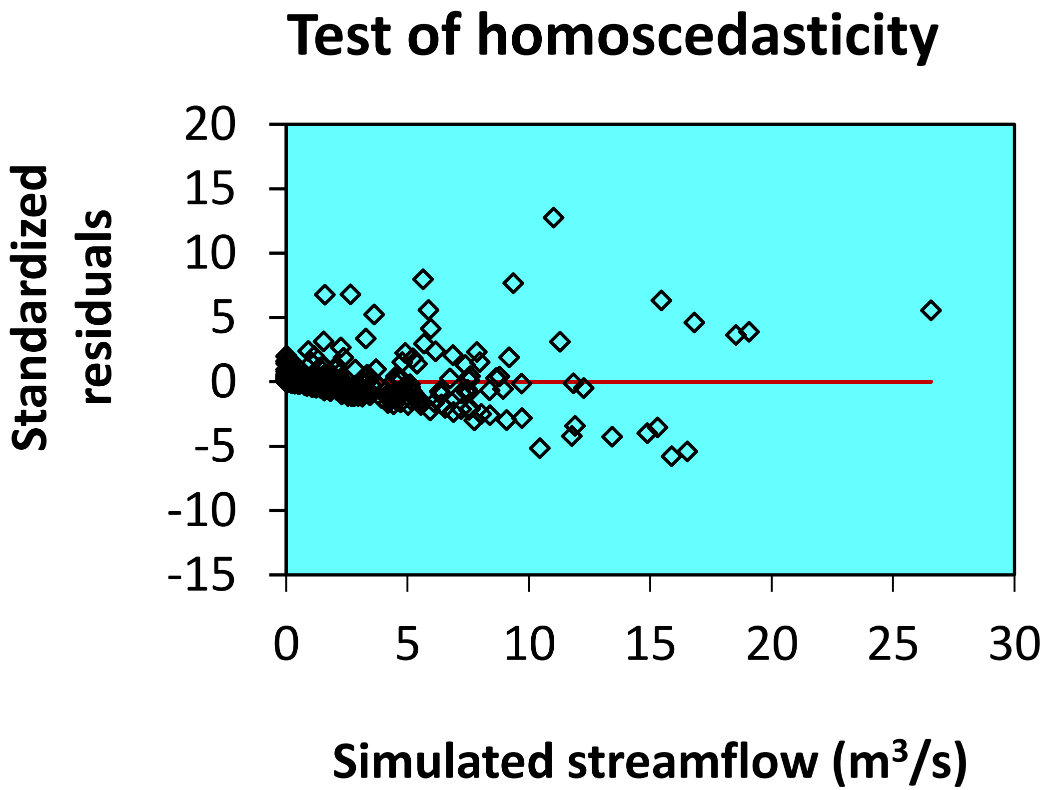
Percentage of observed streamflow data covered by			
95% prediction uncertainty due to parameter uncertainty		95% prediction limits	
Calibration	Validation	Calibration	Validation
11.2	6.1	94.9	94.9

Residual diagnostics





Residual diagnostics



Conclusions

- Parameter uncertainty is higher for CN and EPCO- shows presence of local optima.
- Observed streamflow data covered by parameter uncertainty is 11.2% and 6.1% in model calibration and validation
- Structural uncertainty dominates over parameter uncertainty in streamflow simulation.

Conclusions

- AR(1) model has removed the non-randomness of model residuals.
- Residuals are non-normal and heteroscedastic.
- Heteroscedasticity needs to be considered for better assessment of parameter uncertainty.

Thank you

UA of SWAT model

- SWAT-CUP2 : SUFI-2, GLUE, Parasol and MCMC methods.
- Examples:
  - Setegn et al. (2010) - SUFI-2, Parasol and GLUE
  - Li et al. (2010, 2009) - bootstrap and MCMC methods
  - Ghaffari et al. (2010), Faramarzi et al. (2009) ,

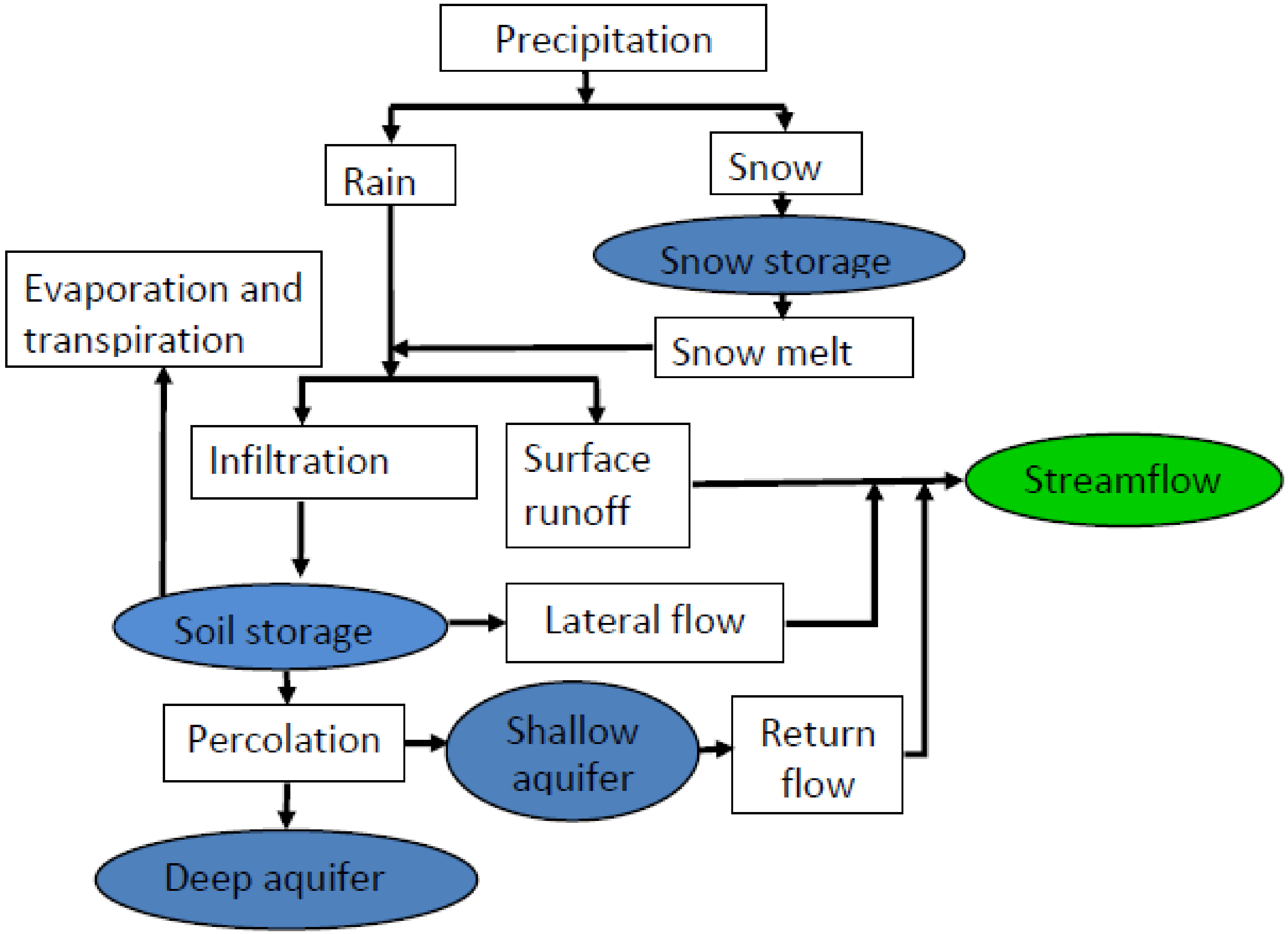
Schuol et al. (2008) - SUFI-2

  - Xie and Zhang (2010)- SDA
  - Zhang et al. (2009)- combined method of GA and BMA
  - Yang et al. (2007a,b)- continuous time AR models with Box-Cox transformation.
- Most of the methods fall under category of UA method-type 1.

Estimation of water balance components

- Infiltration and surface runoff -SCS curve number method.
- Potential evapotranspiration- Penman-Monteith method.
- Lateral subsurface flow- Kinematic storage model.
- Groundwater flow - Computed as return flow to stream from the shallow aquifer.
- Routing - Muskingum method.

SWAT model



Movement of water at the HRU level (Adapted from Neitsch et al., 2005)