EFFECT OF SPATIAL AND TEMPORAL DISCRETIZATIONS ON THE SIMULATIONS USING CONSTANT-PARAMETER AND VARIABLE-PARAMETER MUSKINGUM METHODS

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Introduction

- Flood wave movement is a nonlinear process
- Many models used for catchment runoff simulations still employ linear theory based models for runoff simulations
- FLOOD ROUTING METHODS
- **CONSTANT-PARAMETER MUSKINGUM METHOD (MRM)**

(as used in the SWAT model based on the Muskingum-Cunge (Cunge1969) approach)

- **VARIABLE-PARAMETER MUSKINGUM METHOD (VPMM)**

In focus

The focus of this study is concerned only with one of the component processes, namely, flood routing in main channels using the Muskingum method.

Objective is:

- To explore the impact of spatial and temporal discretizations on the routing simulations of the constant-parameter and the variable-parameter Muskingum methods

CONSTANT-PARAMETER MUSKINGUM METHOD

Frame work of Muskingum Routing Method (MRM)

(as used in the SWAT model based on the Muskingum-Cunge (Cunge1969) approach)

Routing Equation $$Q_2 = C_1 I_2 + C_2 I_1 + C_3 Q_1$$

Parameters

$$K = \frac{1000 L_k}{C_k}$$

$$C_k = \frac{\partial Q}{\partial A} = \frac{5}{3} V_c$$

where L_k and C_k respectively are the channel length (km) and celerity of the flood wave (m/s)

V_c is the channel flow velocity (m/s).

Coefficients

$$C_1 = \frac{-2K\theta + \Delta t}{2K(1-\theta) + \Delta t}$$

$$C_2 = \frac{2K\theta + \Delta t}{2K(1-\theta) + \Delta t}$$

$$C_3 = \frac{2K(1-\theta) - \Delta t}{2K(1-\theta) + \Delta t}$$

θ is weighing factor (0-0.5);

To avoid the numerical instability and negative outflow computation, the following condition is recommended in the SWAT model

$$2K\theta < \Delta t < 2K(1-\theta)$$

The constant parameter-Muskingum method by estimating its two constant parameters using a reference discharge.

$Q_{ref} = (Q_{pave} + Q_b)/2$

$Q_{pave} = (Q_{pinf} + qpout)/2$

Y_{ref} (from Newtorn Raphson method)

If $\Delta t < 2k\theta$, the Muskingum coefficient C_o = - Ve
Generally negative values of coefficients are avoided

For best results, the Δt should be so chosen that $k > \Delta t > 2k\theta$

VARIABLE-PARAMETER MUSKINGUM METHOD (VPMM)

Frame Work of the VPMM Method

The Variable Parameter McCarthy-Muskingum (VPMM) method proposed by *Perumal and Price* [2013]

- directly derived from the Saint-Venant Equations.
- for routing flood waves in semi-infinite rigid bed prismatic channels having any cross-sectional shape/follows either Mannings friction law.
- during steady flow in the channel reach there exist discharge and depth relationship.
- during unsteady flow in the channel reach, the discharge observed at any section has its corresponding normal depth at upstream section.
- Allows the Simultaneous computations of stage hydrograph corresponding to a given inflow/routed hydrograph.

Frame Work of VPMM Method

Saint-Venant equations

Continuity equation

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = 0 \qquad \textbf{(1)}$$

Momentum equation

$$S_f = S_0 - \frac{\partial y}{\partial x} - \frac{v}{g}\frac{\partial v}{\partial x} - \frac{1}{g}\frac{\partial v}{\partial t} \qquad \textbf{(2)}$$

S_f = friction slope;
S_o= bed slope;
$\partial y/\partial x$ = water surface slope;
$(v/g)(\partial v/\partial x)$= convective acceleration;
$(1/g)(\partial v/\partial t)$=local acceleration.
Magnitudes of various terms in eqn. (2) are usually small in comparison with S_o [*Henderson,* 1966*; NERC,* 1975].

Parameter estimation using the VPMM method

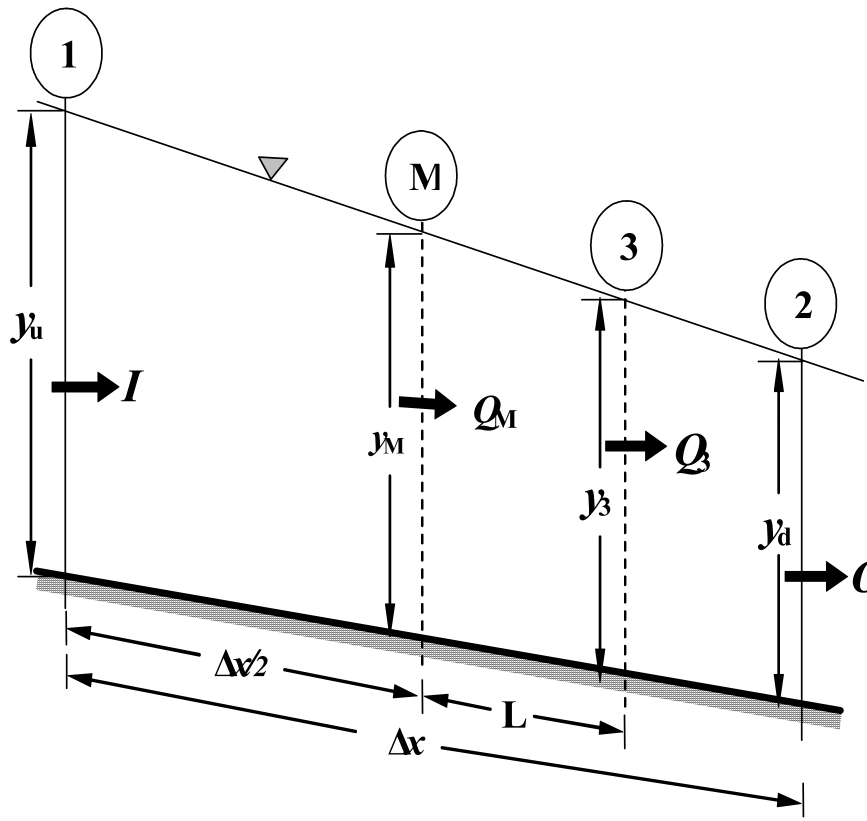


Fig. 1 Definition Sketch of the VPMM Routing Reach

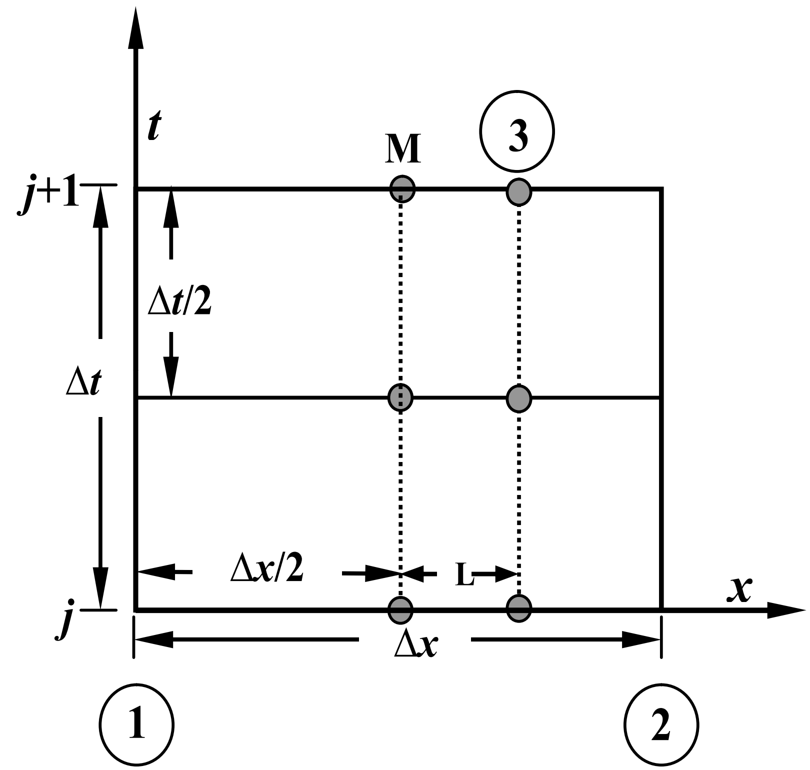


Fig. 2 Numerical grid adopted for VPMM application in synchronization with Fig.1

Parameter estimation using the VPMM method

Muskingum Routing Equation

$$Q_{d,(j+1)\Delta t} = C_1 Q_{u,(j+1)\Delta t} + C_2 Q_{u,j\Delta t} + C_3 Q_{d,j\Delta t}$$

Where

$Q_{d,(j+1)\Delta t}$ The estimated downstream discharges at time $(j+1)\Delta t$

$Q_{u,(j+1)\Delta t}$ The observed upstream discharges at time $(j+1)\Delta t$

$Q_{u,j\Delta t}$ The observed upstream discharges at time $j\Delta t$

$Q_{d,j\Delta t}$ The estimated downstream discharges at time $j\Delta t$

Δt The routing time interval

Coefficients

$$C_1 = \frac{\Delta t - 2.K_{,(j+1)\Delta t}.\theta_{,(j+1)\Delta t}}{\Delta t + 2.K_{,(j+1)\Delta t}.\left(1-\theta_{,(j+1)\Delta t}\right)}$$

$$C_2 = \frac{\Delta t + 2.K_{,j\Delta t}.\theta_{,j\Delta t}}{\Delta t + 2.K_{,(j+1)\Delta t}.\left(1-\theta_{,(j+1)\Delta t}\right)}$$

$$C_3 = \frac{-\Delta t + 2.K_{,j\Delta t}.\left(1-\theta_{,j\Delta t}\right)}{\Delta t + 2.K_{,(j+1)\Delta t}.\left(1-\theta_{,(j+1)\Delta t}\right)}$$

Parameter estimation using the VPMM method

Routing parameters

$$K_{,(j+1)\Delta t} = \frac{\Delta x}{V_{Mo,(j+1)\Delta t}}$$

$$\theta_{,(j+1)\Delta t} = \frac{1}{2} - \frac{Q_{3,(j+1)\Delta t}}{2.S_o.B_{M,(j+1)\Delta t}.c_{Mo,(j+1)\Delta t}.\Delta x}$$

(By neglecting inertial terms)

$$\theta_{,(j+1)\Delta t} = \frac{1}{2} - \frac{Q_{3,(j+1)\Delta t}\left[1 - \frac{4}{9}F^2\left(\frac{PdR/dy}{dA/dy}\right)^2\right]_{M,(j+1)\Delta t}}{2.S_o.B_{M,(j+1)\Delta t}.c_{Mo,(j+1)\Delta t}.\Delta x}$$

$$c = \frac{dQ}{dA} = \left[1 + \frac{2}{3}\frac{P}{B}\frac{dR}{dy}\right]v$$

The governing finite difference equation of the VPMM routing method

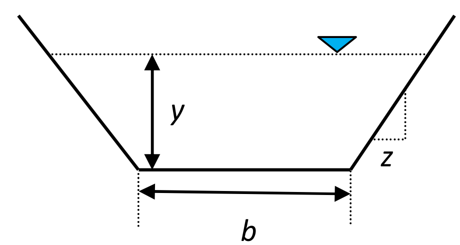
$$\Delta x\left[\frac{Q_{3,(j+1)\Delta t}}{V_{Mo,(j+1)\Delta t}} - \frac{Q_{3,j\Delta t}}{V_{Mo,j\Delta t}}\right] = \Delta t\left[\frac{I_{,(j+1)\Delta t} + I_{,j\Delta t}}{2} - \frac{O_{,(j+1)\Delta t} + O_{,j\Delta t}}{2}\right]$$

Numerical Experiments

Twelve numerical test runs have been conducted in this study by changing space (Δx) and routing time (Δt) intervals to explore the effect of spatial and temporal discretizations on the routing simulations of the constant-parameter and the variable-parameter Muskingum methods.

In this study, a uniform prismatic trapezoidal channel with a bed slope, $\boldsymbol{S_o}$ **= 0.0002,** Manning's roughness coefficient***, n = 0.04*** *and* bed **width of 50m** contained by dykes with a slope ratio (1 V : 5 Horizontal (z)) has been used.

Test Run No.	Space step (km)	Time step (h)
1	40 (Single Reach)	1.00
2	40 (Single Reach)	3.00
3	40 (Single Reach)	6.00
4	40 (Single Reach)	24.00
5	4 (10 Sub-reaches)	1.00
6	4 (10 Sub-reaches)	3.00
7	4 (10 Sub-reaches)	6.00
8	4 (10 Sub-reaches)	24.00
9	1 (40 Sub-reaches)	1.00
10	1 (40 Sub-reaches)	3.00
11	1 (40 Sub-reaches)	6.00
12	1 (40 Sub-reaches)	24.00



Trapezoidal channel

Inflow Hydrograph and Benchmark solution details

Pearson type-III distribution expressed as

$$Q(t) = Q_b + (Q_p - Q_b)\left(\frac{t}{t_p}\right)^{1/(\gamma-1)} \exp\left(\frac{1 - t/t_p}{\gamma - 1}\right)$$

The benchmark solutions were obtained by HEC-RAS model (*USACE*, 2010) by routing the inflow hydrograph for a reach length of 40km in prismatic Trapezoidal channel using the Space step = 1000m and Time step = 300sec

**Flow characteristics

Q_b = 100 m^3/sec

Q_p = 3000 m^3/sec

T_p = 192 h (8 days),

Shape factor $\gamma = 1.05$

Results and Discussions

Table 1 Summary of performance criteria showing reproduction of pertinent characteristics of the HEC-RAS results by the VPMM method for routing in Trapezoidal channel reaches using spatial and temporal discretization on the simulations.

Test Run No.	Space step (km)	Time step (h)	μ_q (%)	μ_y (%)	Discharge Routing				Stage Computation		
					$EVOL$ (%)	η_q (%)	q_{per} (%)	t_{pqer} (h)	η_y (%)	y_{per} (%)	t_{pyer} (h)
1	40	1.00	0.63	0.26	-0.00029	100.00	0.02	0.00	100.00	0.28	2.00
2	40	3.00	0.64	0.26	-0.00086	94.95	-4.30	3.00	93.36	-1.14	6.00
3	40	6.00	0.64	0.2	-0.00262	75.82	-13.73	4.00	66.62	-4.00	6.00
4	40	24.00	1.36	0.86	-0.06547	-22.23	-48.67	3.00	-69.74	-13.96	5.00
5	4	1.00	0.63	0.26	-0.00031	100.00	0.02	0.00	100.00	0.15	2.00
6	4	3.00	0.64	0.26	-0.00088	94.66	-4.45	3.00	92.86	-2.26	6.00
7	4	6.00	0.64	0.2	-0.00171	73.56	-15.24	4.00	64.50	-8.22	6.00
8	4	24.00	1.36	0.86	-0.00572	-9.89	-61.89	5.00	-51.24	-37.75	7.00
9	1	1.00	0.63	0.26	-0.00030	100.00	0.02	0.00	100.00	0.14	2.00
10	1	3.00	0.64	0.26	-0.00089	94.66	-4.46	3.00	92.85	-2.35	6.00
11	1	6.00	0.64	0.2	-0.00168	73.55	-15.28	4.00	64.62	-8.49	6.00
12	1	24.00	1.36	0.86	-0.00572	-9.11	-62.10	5.00	-48.35	-38.49	7.00

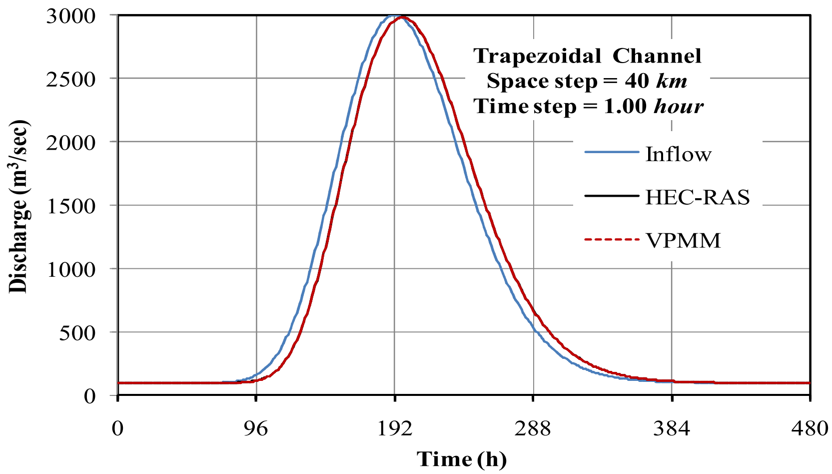


Figure 1. Typical simulated discharge hydrograph of the VPMM method for a space step of 40*km* and a time step of 1.00*hour* in a trapezoidal channel.

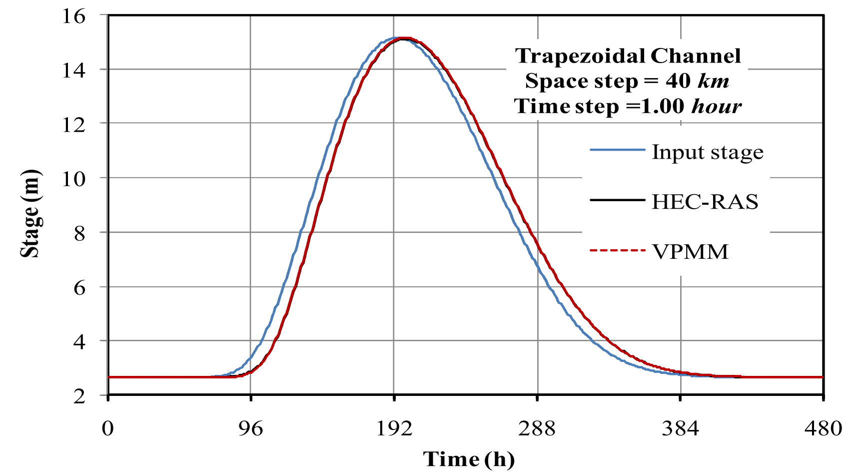


Figure 2. Typical computed stage hydrographs of the VPMM method for a space step of 40*km* and a time step of 1.00*hour* in a trapezoidal channel.

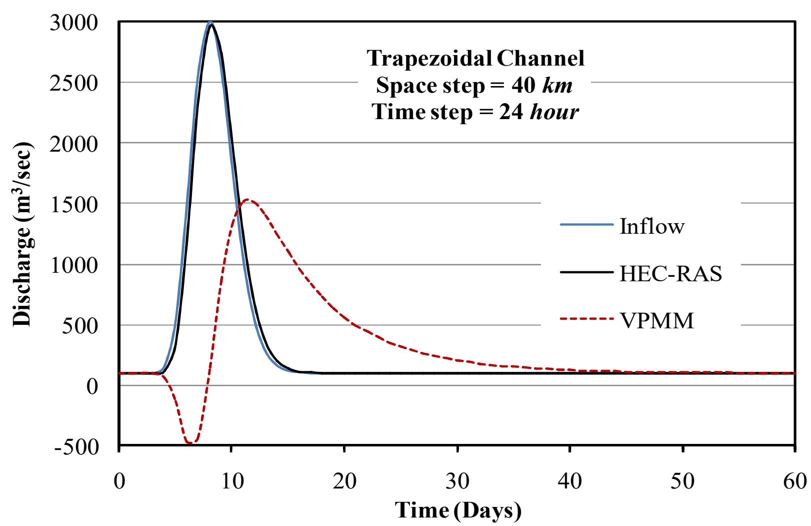


Figure 3. Typical simulated discharge hydrograph of the VPMM method for a space step of 40*km* and a time step of 24.00*hour* in a trapezoidal channel.

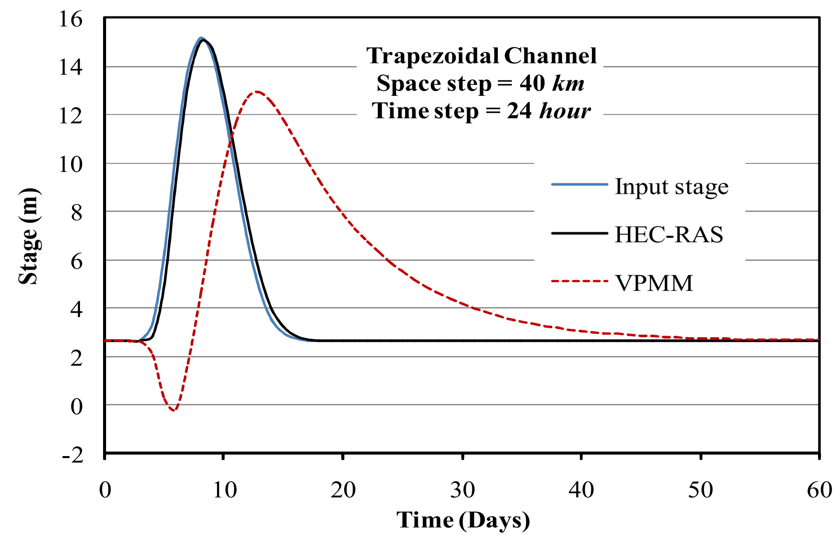


Figure 4. Typical computed stage hydrographs of the VPMM method for a space step of 40*km* and a time step of 24.00*hour* in a trapezoidal channel.

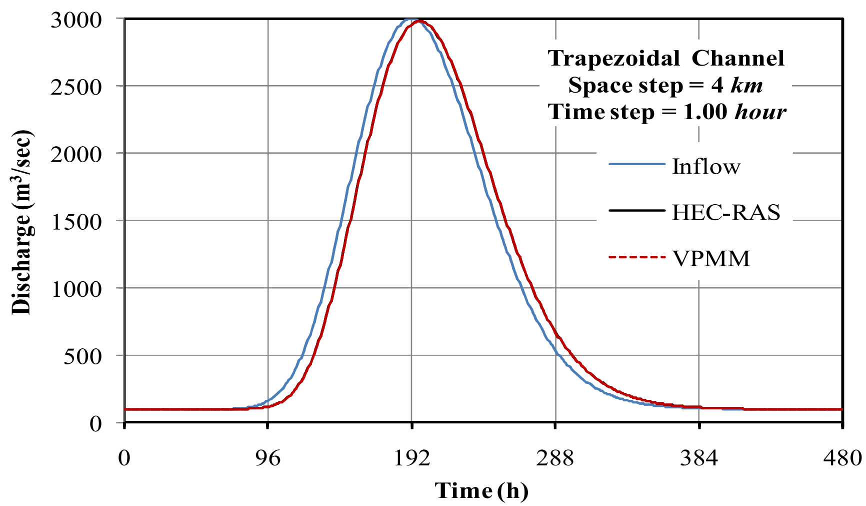


Figure 5. Typical simulated discharge hydrograph of the VPMM method for a space step of 4*km* and a time step of 1.00*hour* in a trapezoidal channel.

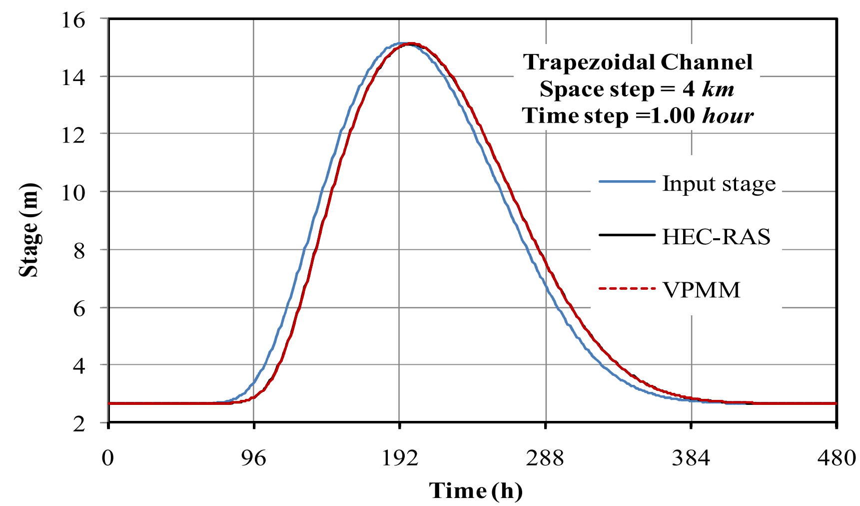


Figure 6. Typical computed stage hydrographs of the VPMM method for a space step of 4*km* and a time step of 1.00*hour* in a trapezoidal channel.

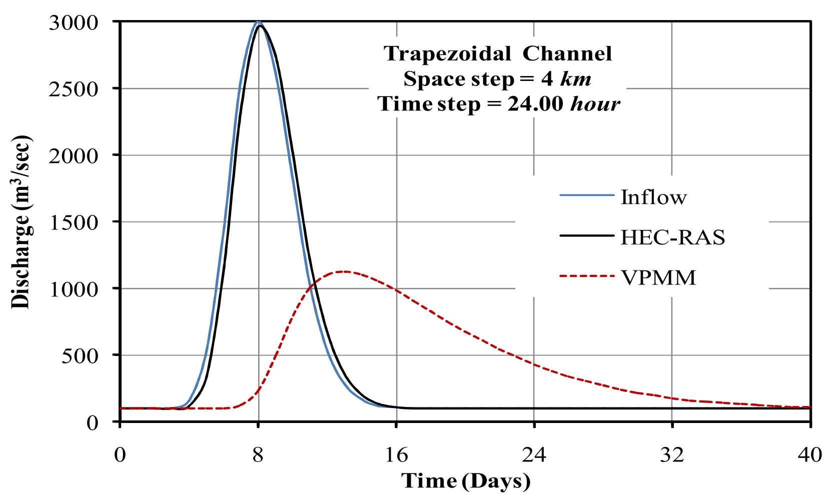


Figure 7. Typical simulated discharge hydrograph of the VPMM method for a space step of 4*km* and a time step of 24.00*hour* in a trapezoidal channel.

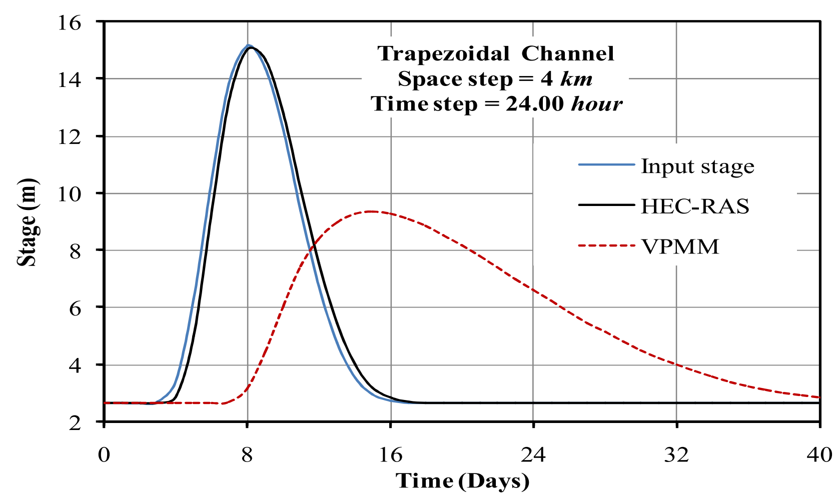


Figure 8. Typical computed stage hydrographs of the VPMM method for a space step of 4*km* and a time step of 24.00*hour* in a trapezoidal channel.

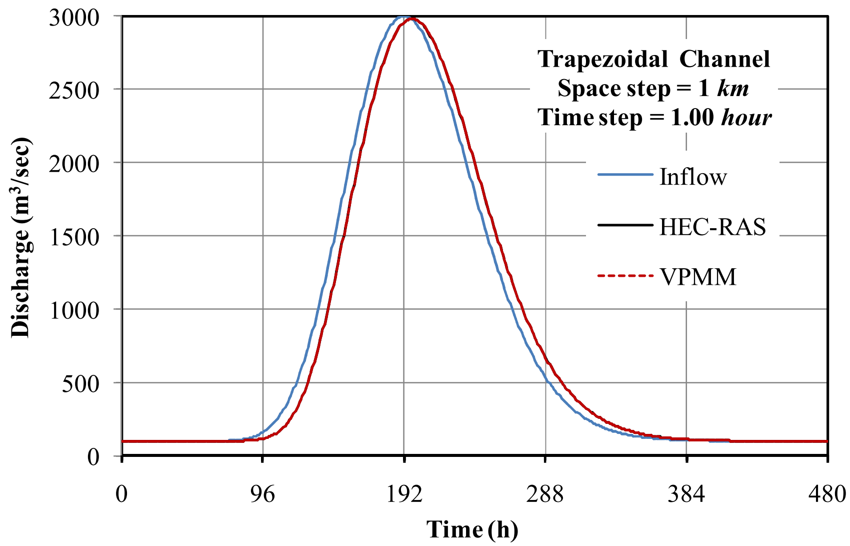


Figure 9. Typical simulated discharge hydrograph of the VPMM method for a space step of 1*km* and a time step of 1.00*hour* in a trapezoidal channel.

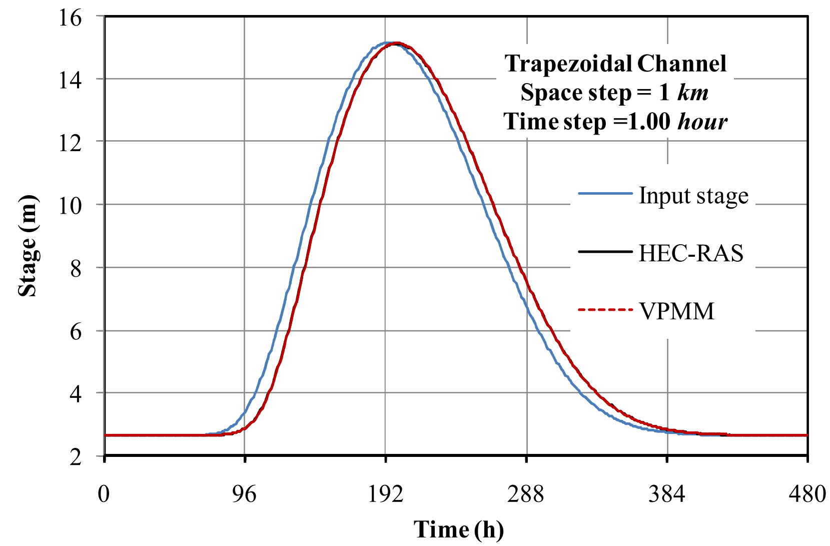


Figure 10. Typical computed stage hydrographs of the VPMM method for a space step of 1*km* and a time step of 1.00*hour* in a trapezoidal channel.

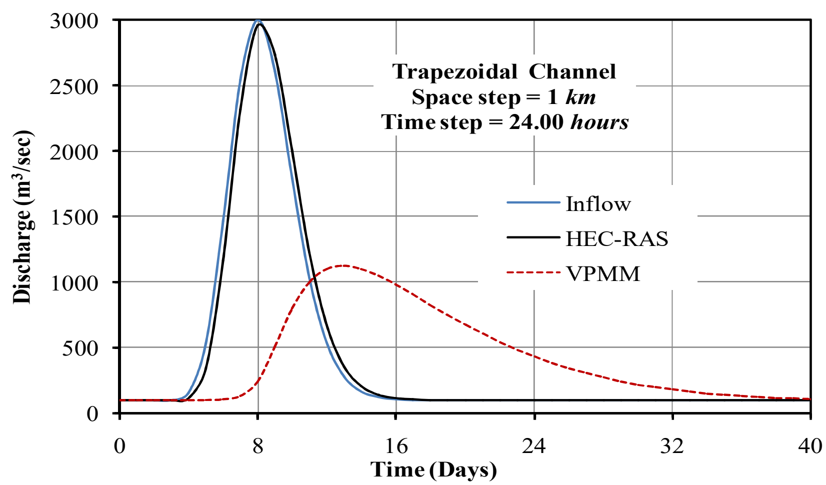


Figure 11. Typical simulated discharge hydrograph of the VPMM method for a space step of 1*km* and a time step of 24.00*hour* in a trapezoidal channel.

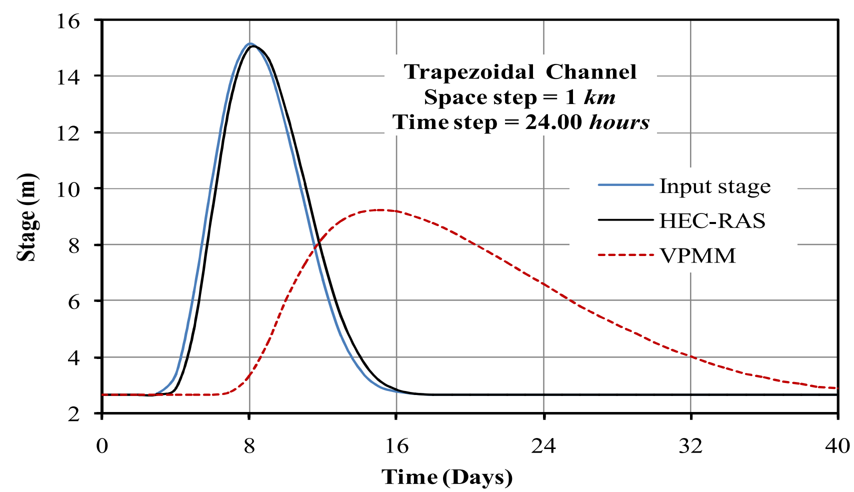


Figure 12. Typical computed stage hydrographs of the VPMM method for a space step of 1*km* and a time step of 24.00*hour* in a trapezoidal channel.

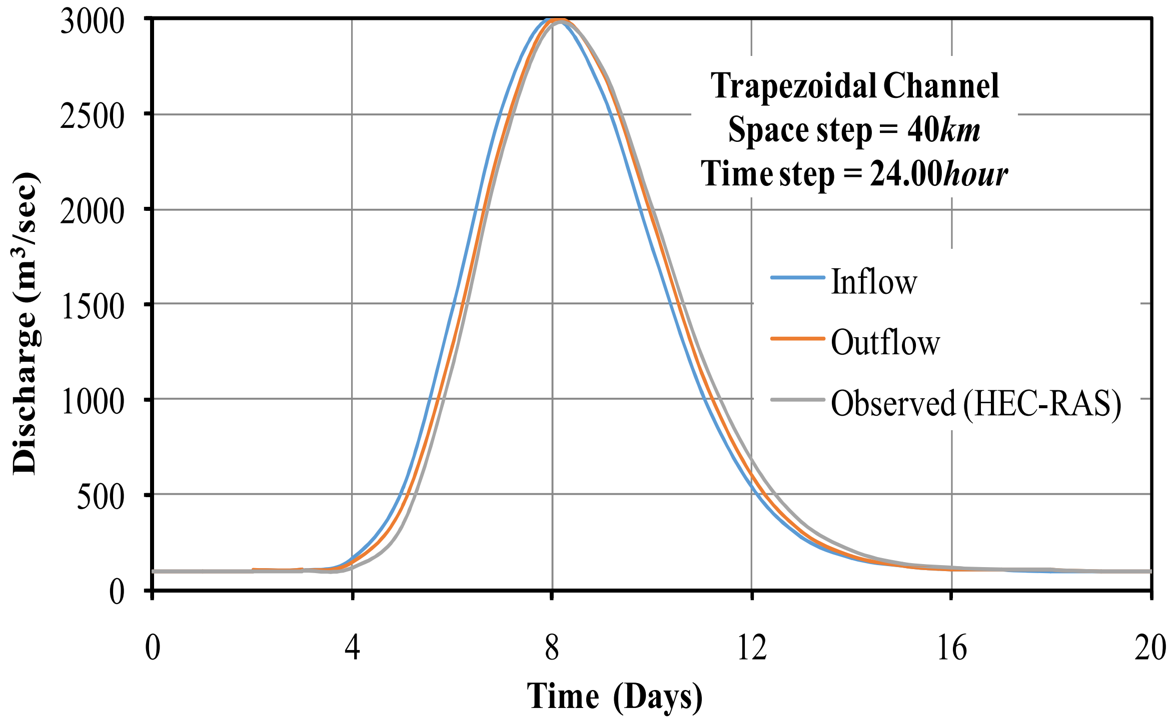


Figure 13. Typical computed discharge hydrographs of the constant-parameter method for a space step of 40*km* and a time step of 24.00*hour* in a trapezoidal channel using reference discharge

Conclusions

- A preliminary investigation carried out with the considered objective shows that the routing solution obtained using a longer routing time interval induces significant numerical diffusion of the routed hydrograph leading to over attenuation of the inflow flood peak and, thereby, resulting in poor reproduction of the benchmark solution.

THANK YOU