Analytic Element Method (AEM) and its Relevance with Subbasin/HRU concept of SWAT for potential integration of AEM based simple ground water model

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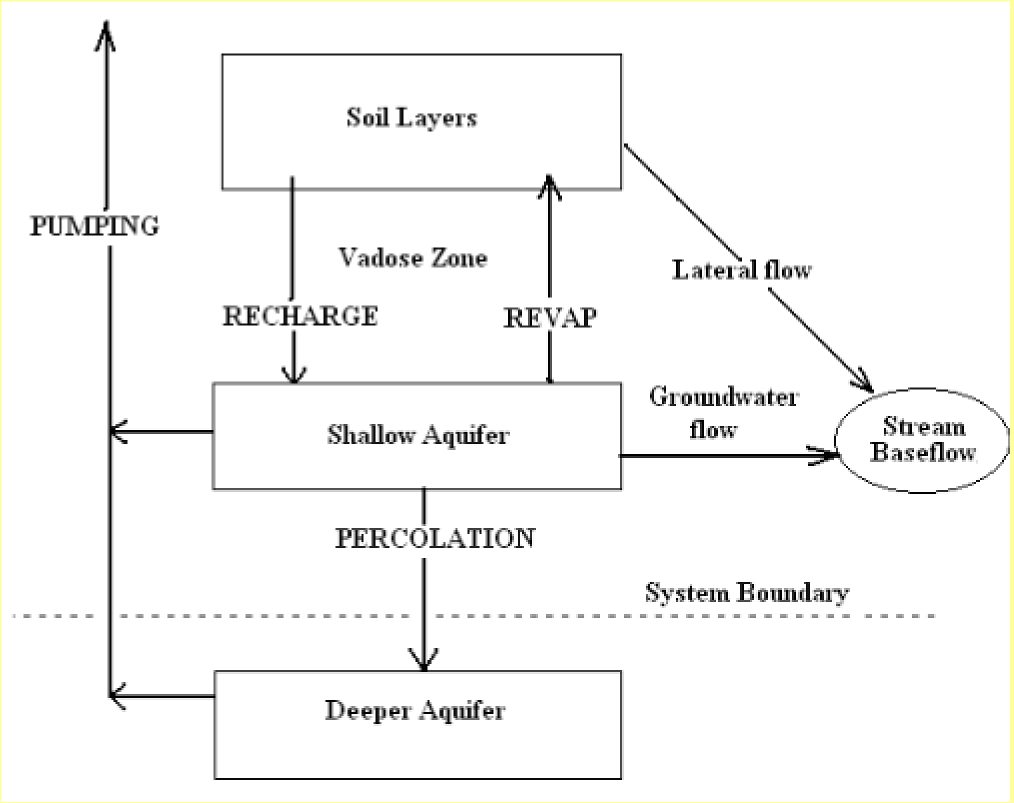
Outline

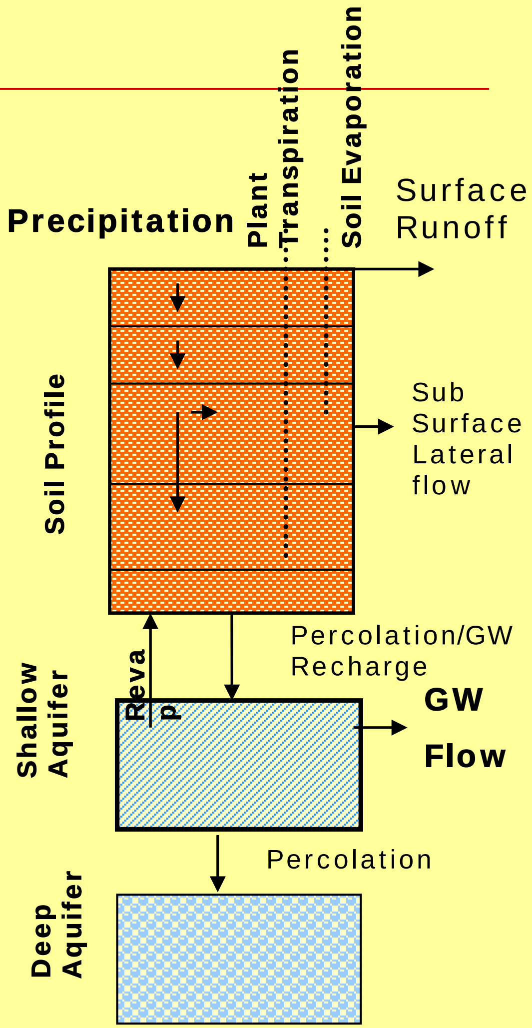
- Introduction
- SWAT GW Model
- Need for study
- Solution methods for solving GW flow problems
- Analytic Element Method and its Assumptions
- Principle of superposition
- Types and representation of Analytic Elements
- Methodology
- Example
- Advantages and limitations of AEM
- Applications of AEM

Introduction

- Streams and Lakes interact with adjacent aquifers, creating complex flow paths in hydrologic cycle.
- Better quantification of water balance components and improved characterization of surface (SW) and ground water (GW) interaction help in improving water resource management
- Several numerical models (FDM/FEM) for ground water modelling is used and coupled with SW model for understanding the SW-GW Interaction.
- This study attempts to integrate Soil and Water Assessment Tool (SWAT) with **Analytic Element Method (AEM)** for ground water modelling

SWAT Ground Water component

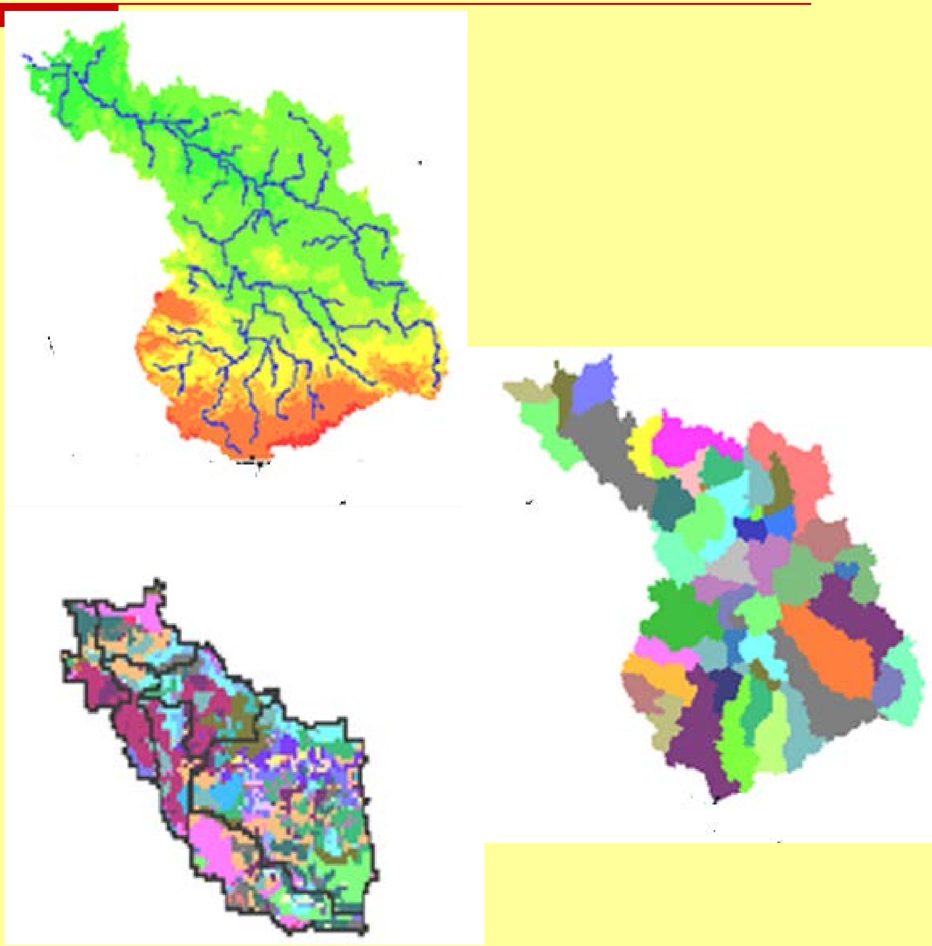




Source: Vazquez-Amábile and Engel (2005)

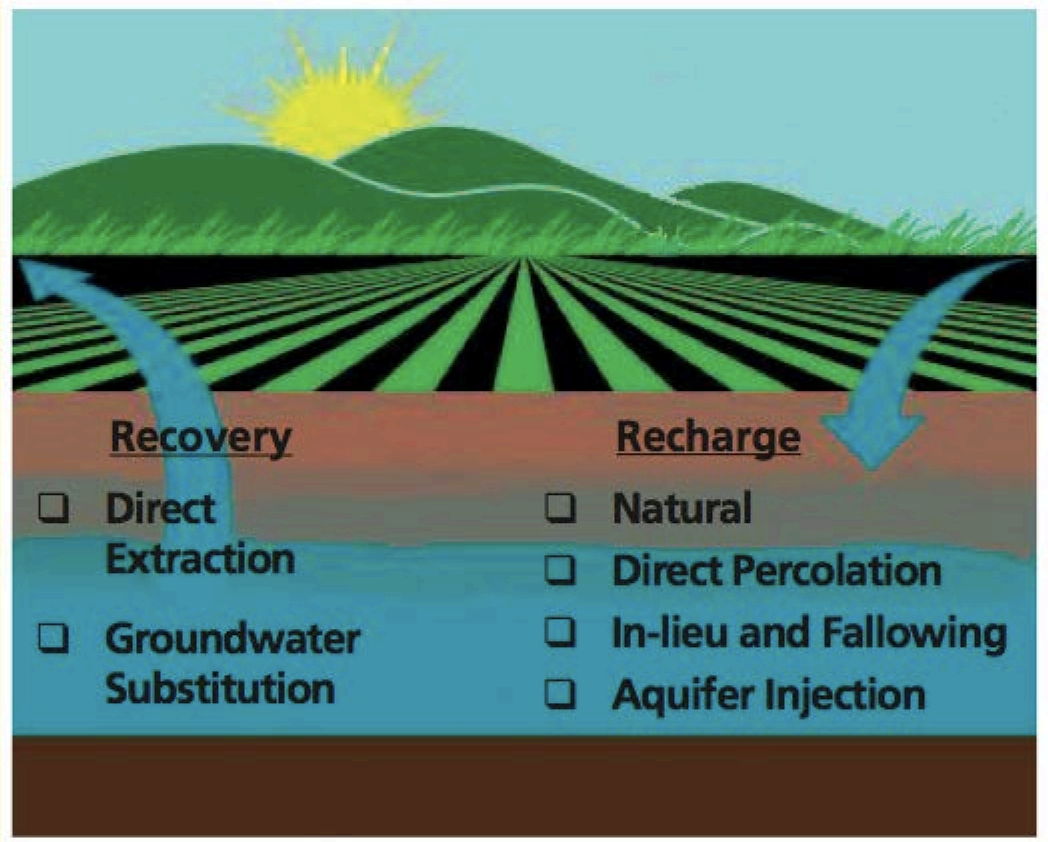
Limitations of SWAT GW model

- SWAT considers each HRU and sub-basins as separate 1-D unit - interactions between them is not considered
- Spatial locations of each HRU - sub-basins are not considered
- Lacks in simulating the distribution and dynamics of GW levels and recharge rates for watersheds



Need for this study

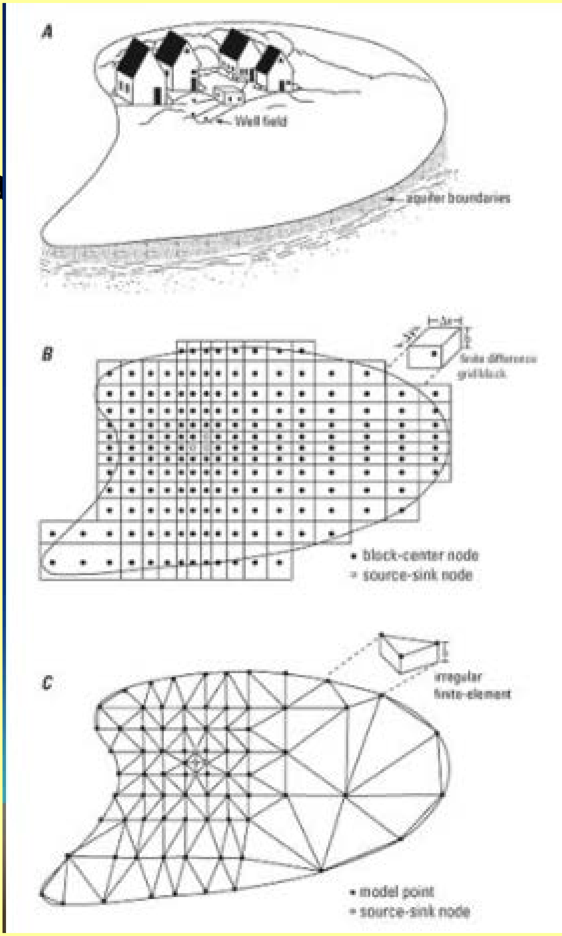
- Conjunctive use of SW-GW resources
- SW and GW interconnected
- Studies are limited in Coupling SW-GW systems
- SWAT coupled with AEM based GW model



Source: Dudley & Fulton (2005)

Solution Methods for Solving Ground Water Flow Problems

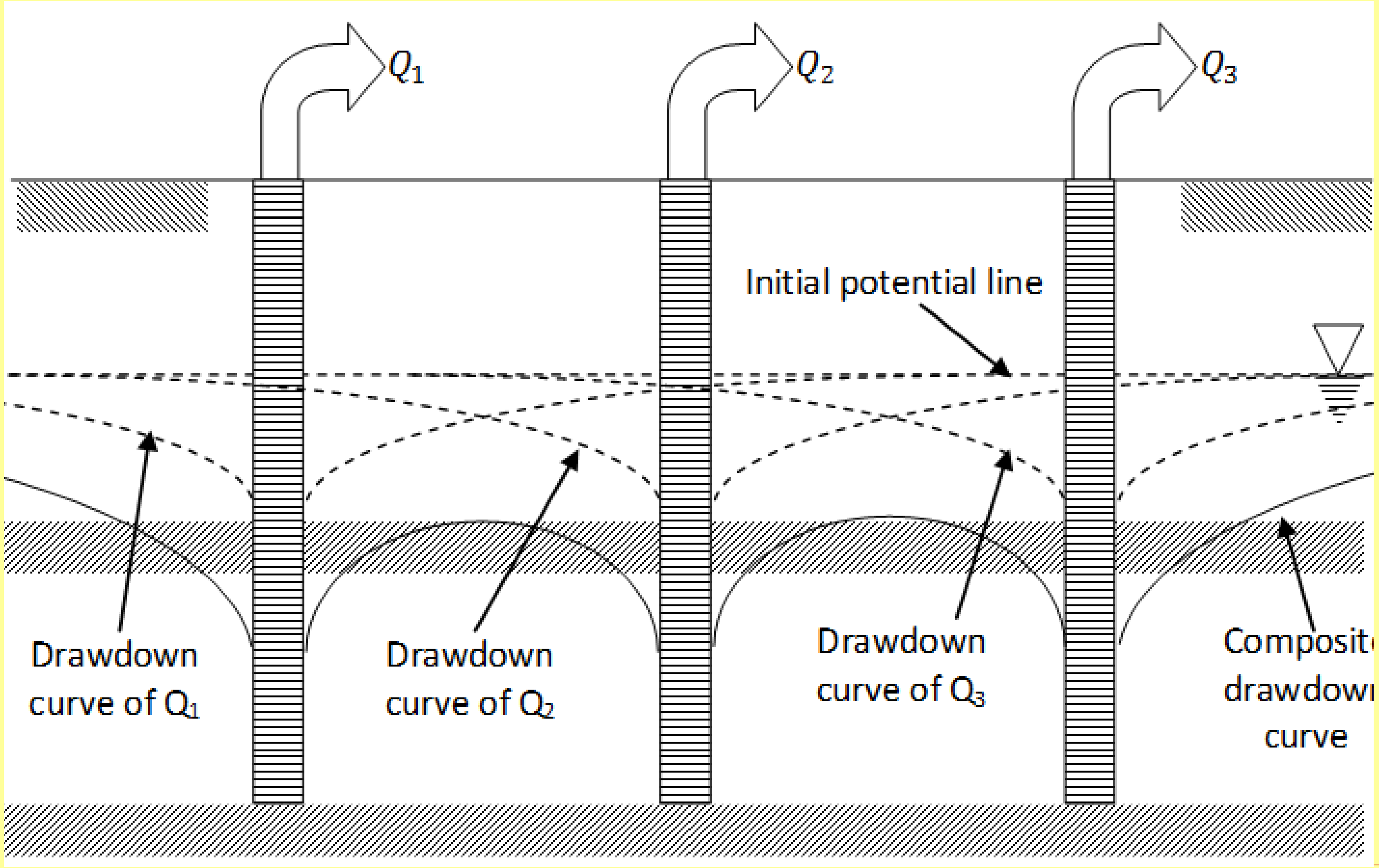
- In order of increasing Complexity :
  - Analytical Solutions : h=f(x,y,z,t) Eg : Theis Equa
  - **Analytic Element Method (AEM)**
  - Numerical solutions : Finite Difference Method (FDM),Finite Element Method (FEM)
- Each solves the governing equation of GW flow and storage in a different approaches.
- The assumptions and applicablility of the methods differs.



Analytic Element Method (AEM)

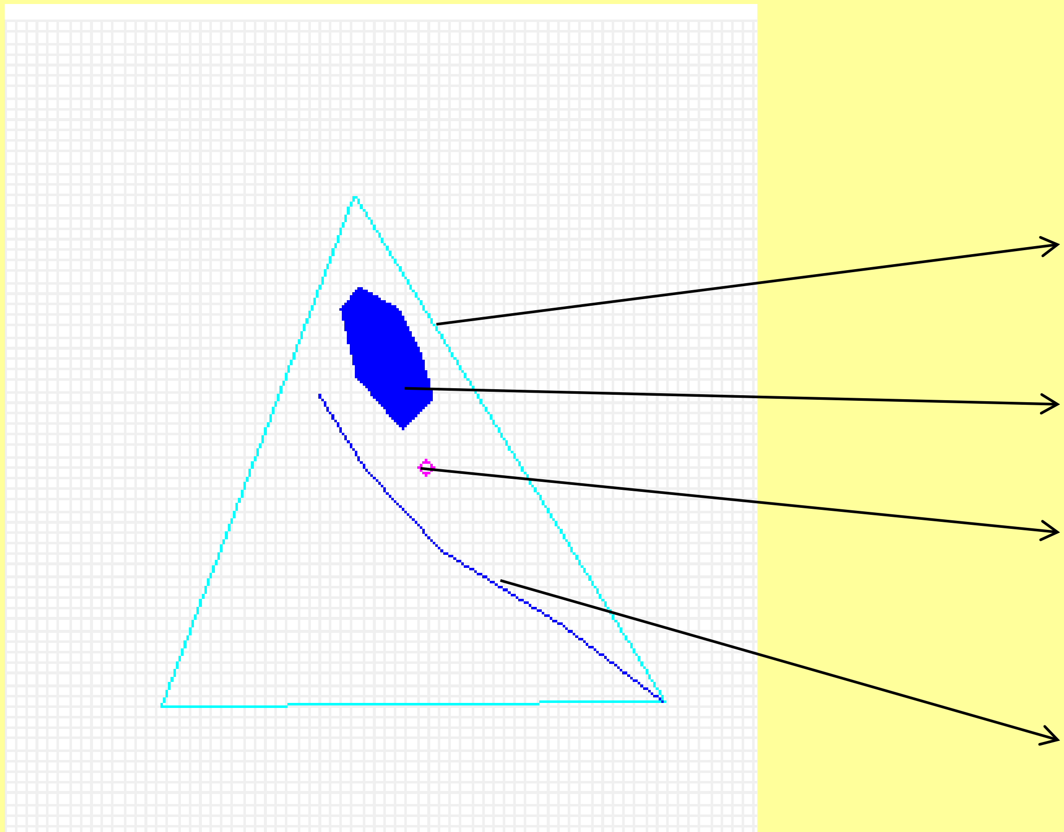
- AEM (Analytic Element method) - proposed by Otto Strack – potential theory
- AEM works on superposition of analytic functions called as analytic elements within a grid independent domain
- Each function (elements) can be modelled individually and super imposed based on linearity of laplace equation to obtain full description of the aquifer.
- Appropriately choosing discharge potentials, rather than piezometric heads, the AEM becomes applicable to both confined and unconfined flow conditions as well as to heterogeneous aquifers

How does principle of superposition work?



e.g.Theis Solution

Type and Representation of Analytic Elements



Area Element (Uniform Recharge)

Area Element (Lake)

Point Element (Well in uniform flow)

Line Element (River)

Principle of Superposition in 1D plane

- Consider GW flow in an infinite strip to a horizontal equipotential
- Thickness : H
- Length of Streambed L
- Two sections of constant recharge : Recharge strip to left N1 and right N2

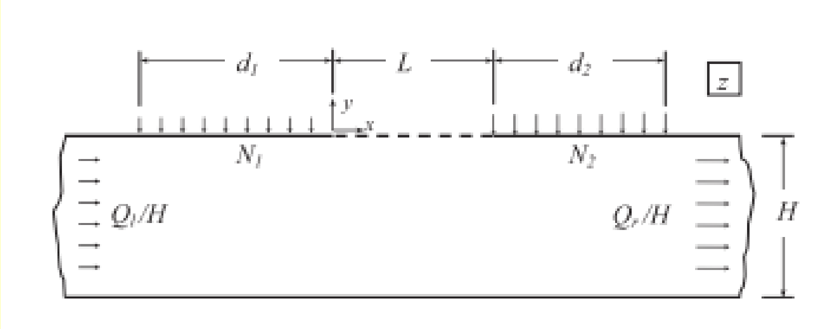


Figure : SW-GW interactions in the vertical plane

$x = -\infty$, Uniform specific discharge = Q_L/H and

$x = \infty$, Uniform specific discharge = Q_R/H

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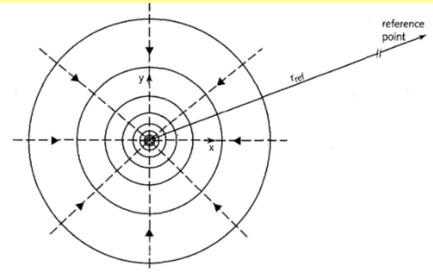
- Solutions to above problem : superposition of 2 recharge strips and solutions containing uniform flow
- GW flow (Laplace equation) expressed as complex potential function

$$\Omega = \Phi + i\Psi$$

- Whereas $\Omega(z)$ = analytic function of a complex coordinate z=x+iy
- Specific discharge potential Φ =kφ , Ψ =Stream function
- Complex discharge potential, $W = -\frac{d\Omega}{dx} = q_x - iq_y$; qx and qy are x, y components of specific discharge vector

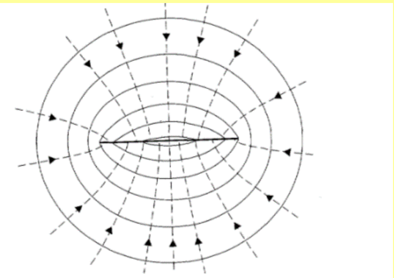
Point Source/Sink (Well Element)



- Withdraw/supply water
- **Input** : Q, R and its location, Extraction or injection (well) rate
- Losing Streams : Radius (+),discharge (-) ; vice versa for gaining streams
- Dirichlet, Neumann Boundary Condition used
- $\Phi = \Re\Omega = \frac{Q}{4\pi}\ln|(x-x_w)^2 + (y-y_w)^2| + \Phi_{ref}$; $\Psi = \Im\Omega = \frac{Q}{2\pi}\arctan\left(\frac{y-y_w}{x-x_w}\right) = \frac{Q}{2\pi}\theta$

$$\Omega = \frac{Q}{2\pi}\ln(z - z_w) + \Phi_{ref}$$

Line Source/Sink
(Streams)



- Infinite number of point sinks (wells) along a straight line
- Inject or extract a net amount of water from the aquifer
- Losing streams : line sink with a negative sink density ; vice versa for gaining streams
- **Input** :
  - Head at beginning and end of a line segment, the resistance, the depth and the width of the river , SW head condition (stage), bottom elevation, width and conductance of underlying bed material

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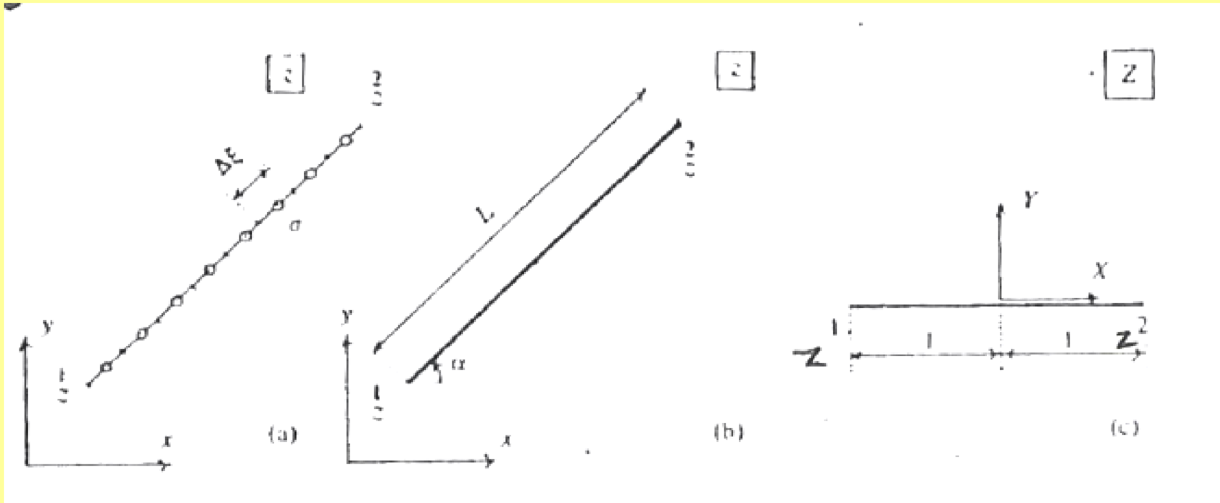
$$\Omega = \int_L \frac{\sigma_{lin}}{2\pi} \ln(z - \delta)\, d\delta + \Phi_{ref}$$

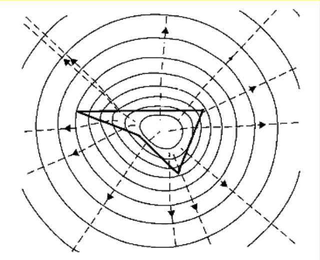
$$\Omega = \frac{\sigma L}{4\pi}\left\{(Z+1)\ln(Z+1) - (Z-1)\ln(Z-1) + 2ln\left[\frac{1}{2}(z_2 - z_1)\right] - 2\right\}$$

$$Z = \frac{z - \frac{1}{2}(z_1 + z_3)}{\frac{1}{2}(z_2 - z_1)}$$

$$\Phi = \Re\Omega = \frac{\sigma L}{4\pi}\left\{-X \ln\left|\frac{Z-1}{Z+1}\right| + Y\theta + \ln|Z^2 - 1|\, 2\ln\frac{L}{2} - 2\right\}$$

$$\Psi = \Im\Omega = \frac{\sigma L}{4\pi}\{-X(\theta_2 - \theta_1) + \theta_1 + \theta_2 + 2\alpha\}$$ (Eg: Along the X axis and Y=0)





Area Source/Sink

- Analytic, non-harmonic functions utilized for representing extraction or infiltration over the area bounded by polygons
- By Combination Of Distributed Doublets And Distributed Line Sinks
- Input : location of the lake boundary, uniform recharge/leakage

$$\Omega_c = \sum_{j=1}^{n} \Omega_j^{\,c} + \frac{Q_o}{2\pi}^{\,c} \ln\left(z - z_1^c\right)$$

$$\Phi i = \left\{\frac{1}{2}\Upsilon\left[(x - x_o{}^c)\cos\alpha_1 + (y - y_o{}^c)\sin\alpha_1\right]^2\right\} + \Omega_c$$

Complex potential, Boundary conditions and the solution

- All potential represent one particular feature in the aquifer
- Complex potential : $\Phi = \Phi_c + \sum_{ne} \Phi e + \sum_{nw} \Phi w + \sum_{nr} \Phi r + \sum_{np} \Phi p + \sum_{nl} \Phi l + \sum_{ni1} \Phi i1 + \sum_{ni2} \Phi i2 + C$
- Complex potential where it is known is noted as U and the parameters are unknown is noted as V
- Above equations written as

$$\Phi\,[\varphi(x_c,y_c)] = U(x_c,y_c) + V(x_c,y_c)$$

- $V(x_c,y_c) = \sum_{nl} \wedge_i A_i(xc, yc) + C$

where $\wedge_i$ = unknown strength or intensity like unknown infiltration from lake ; i=no of elements with unknown parameter A_i = geometry of the element

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- Equations derived from special case of Girinski potential

$$\Phi G = \int_{Z1}^{Z2} (k(z)(\emptyset - z)dz)$$

- For Unconfined aquifer

$$\Phi = \frac{1}{2}kh^2 = \frac{1}{2}k\,(\phi)^2 \;, \phi \leq H$$

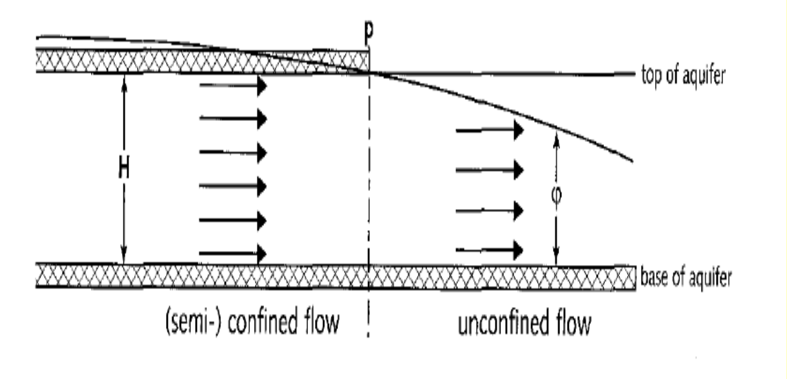
- For confined aquifer

$$\Phi = kH(\phi) - \frac{1}{2}kH^2 \quad \phi \geq H$$

- Equations valid :

$$\nabla^2\Phi = 0 \; or \; \nabla^2\Phi = 0; \nabla^2\psi = 0$$

$$\frac{\partial\psi}{\partial x} = -\frac{\partial\varphi}{\partial y} \text{ and } \frac{\partial\psi}{\partial y} = \frac{\partial\varphi}{\partial x}$$



Examples : No flow condition, uniform flow condition, Vertical inflow of water in an infinite aquifer

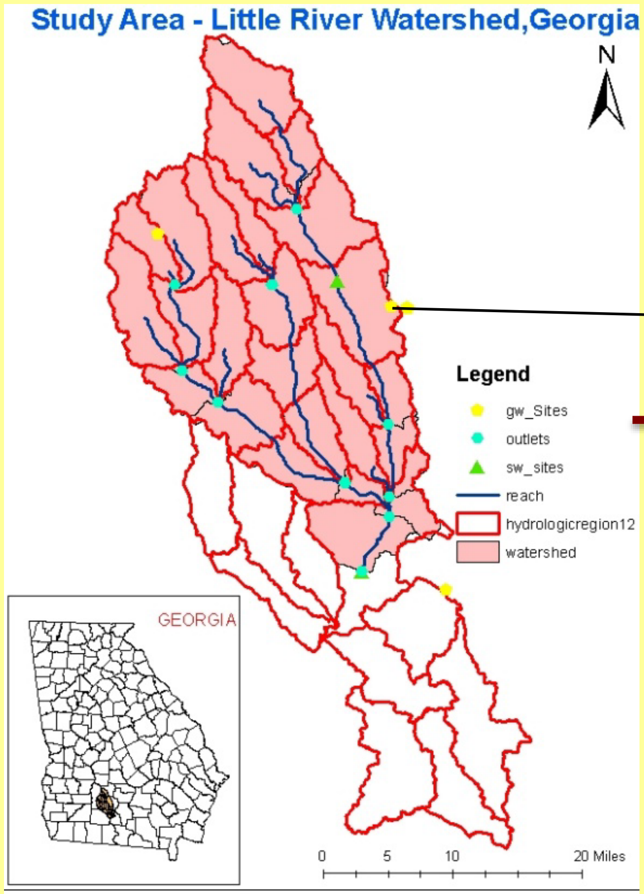
Applications of AEM

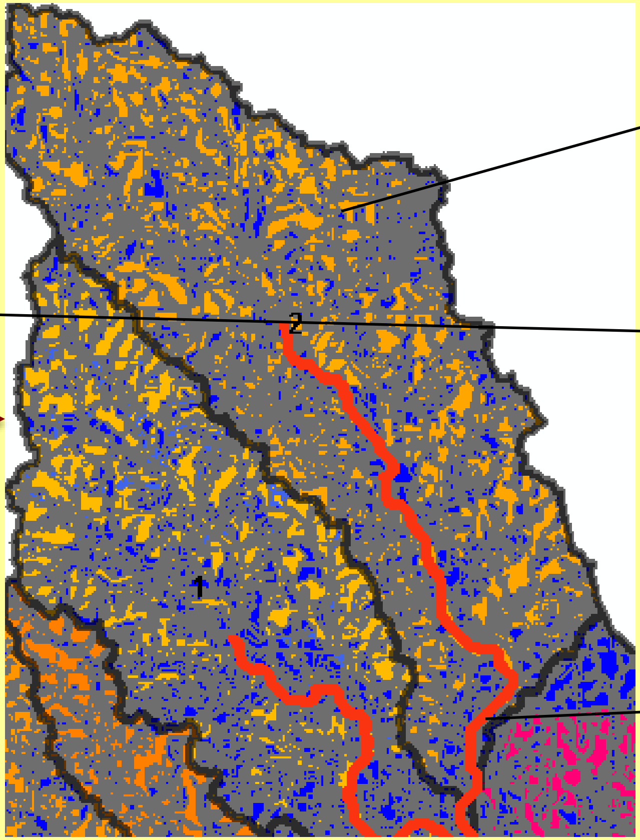
- Where there exists many surface water features or other structures (wells) influence/determine the GW flow over the AOI/ joint effects with corresponding elements
- Sharp variation in the GW levels , Eg : around wells or well field as the method can model both local and regional effects rather accurately
- Well head and capture zone delineation
- Local and Regional GW modelling

Input data to AEM

- Base Elevation - Soil borings
- Thickness of aquifer - Soil borings
- Hydraulic conductivity – Pump test, slug test
- Porosity – from literature
- Boundary conditions

SWAT Input to AEM Input



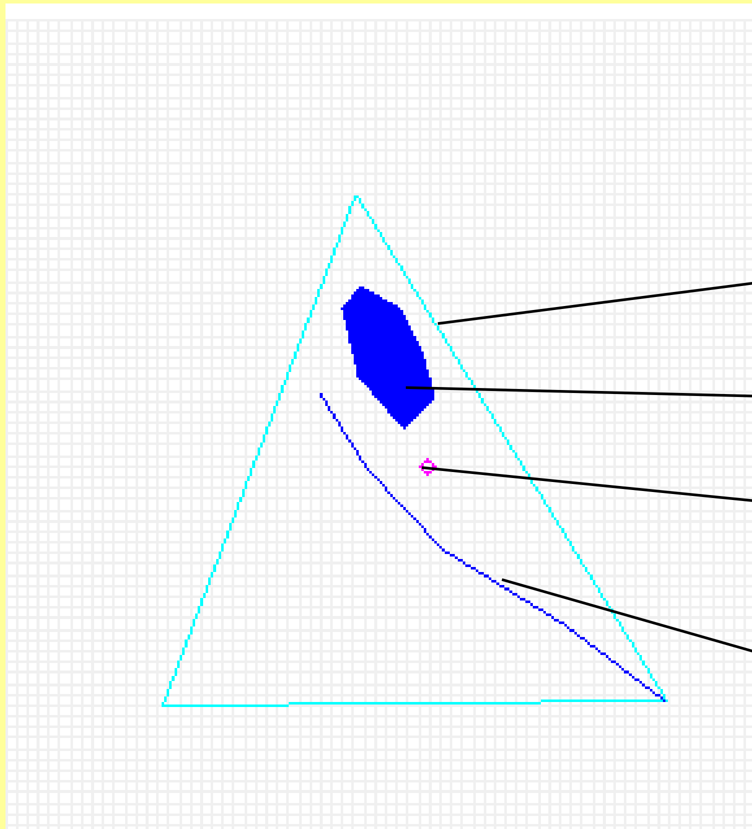


HRU will be treated as Area Element

Well will be treated as Point Element

River will be treated as Line Element

Example in VISUAL AEM



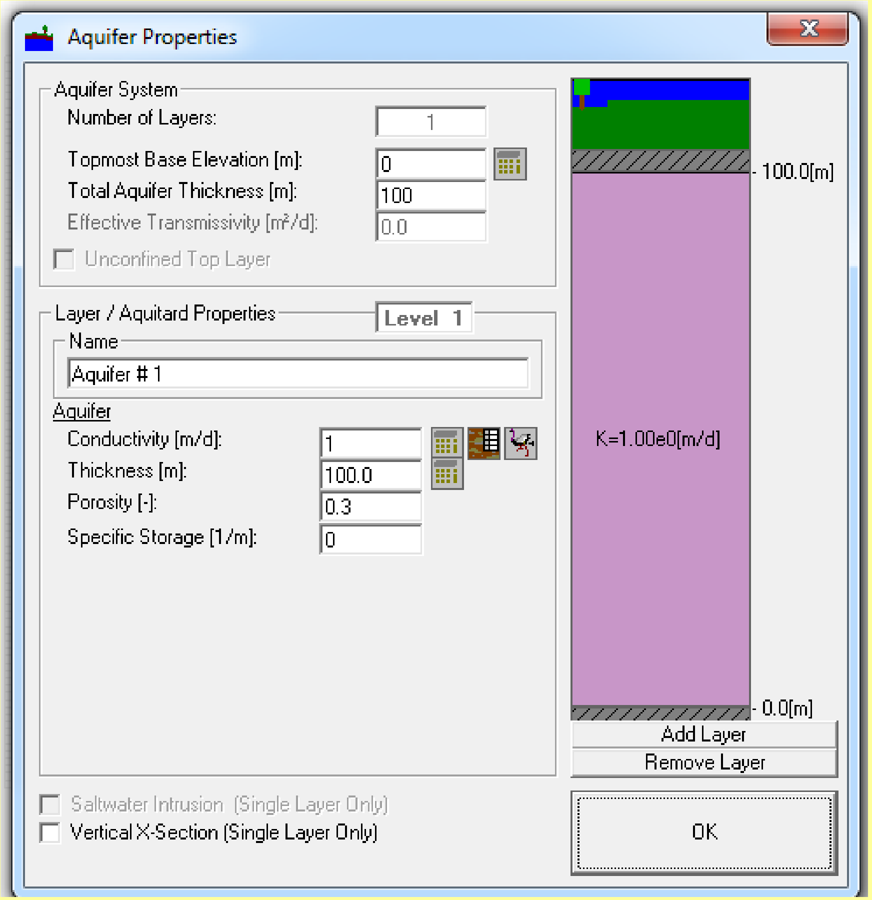
Area Element (Uniform Recharge; HRU)

Area Element (Lake)

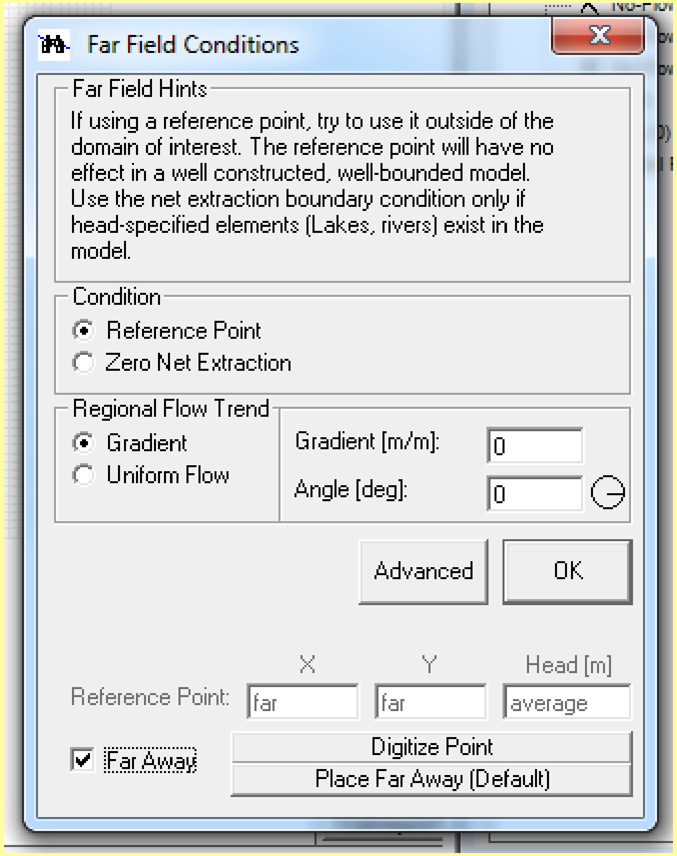
Point Element (Well in uniform flow)

Line Element (River)

Step 1 : Enter Aquifer Properties

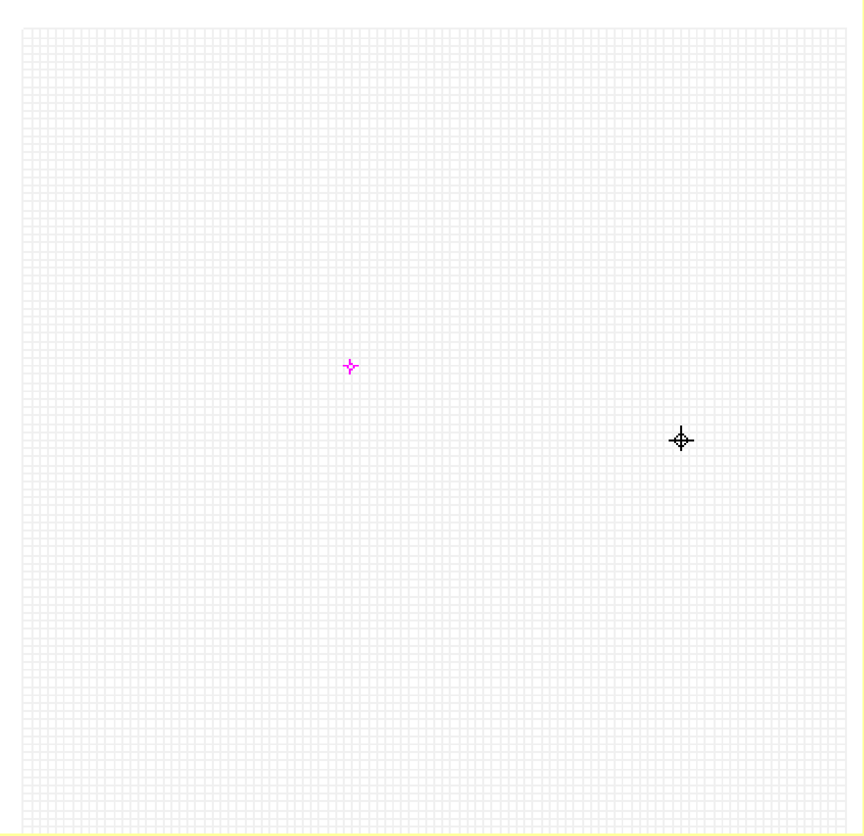


Step 2 : Enter Boundary Conditions



Step 3 : Element Generation

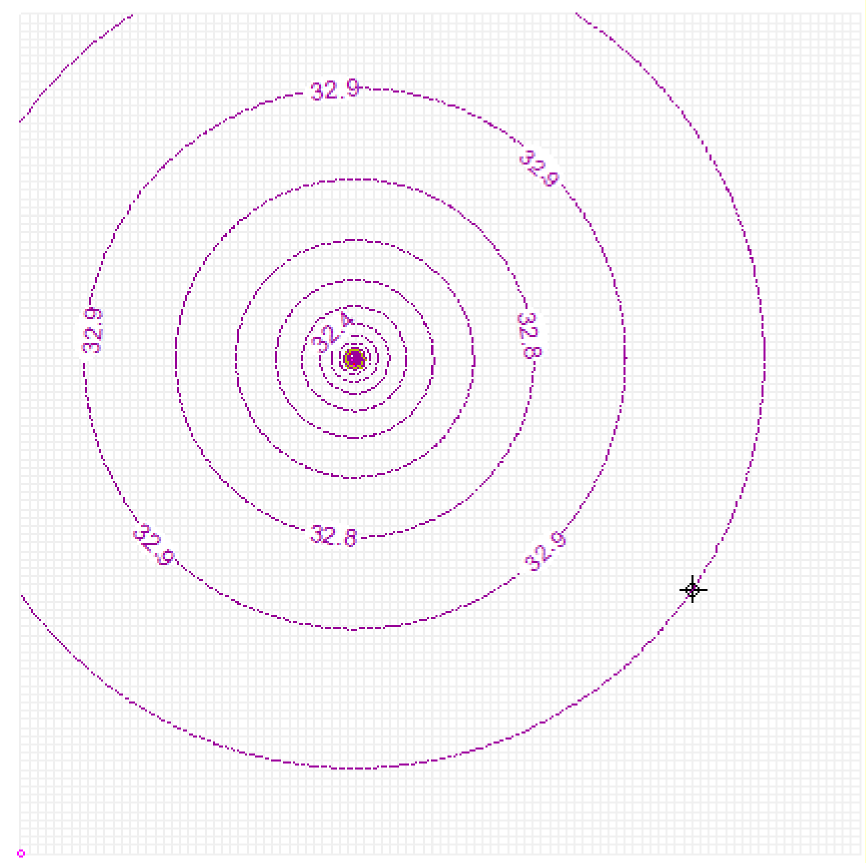
Well Element



Input :

Extraction Well
Discharge : 50 m^3/d
Well radius :0.1 m
Well location :
x = 39.8109
y = 58.956

Head Distribution and Flow vectors



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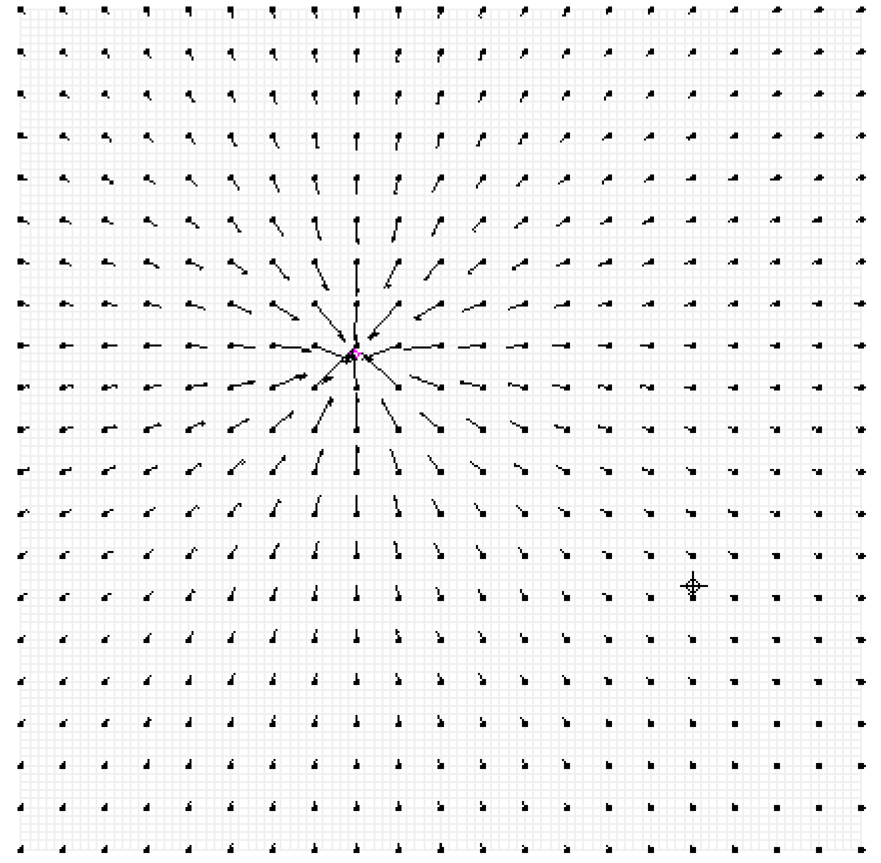

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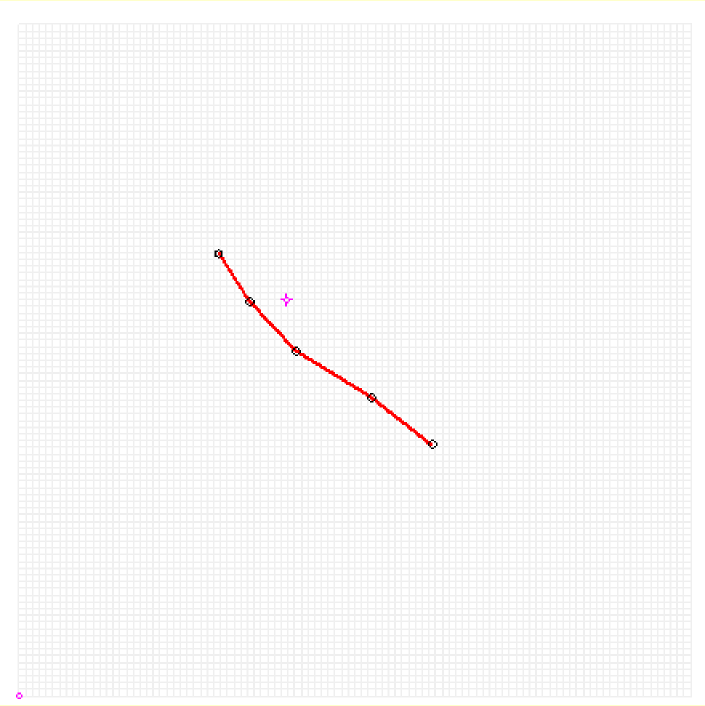
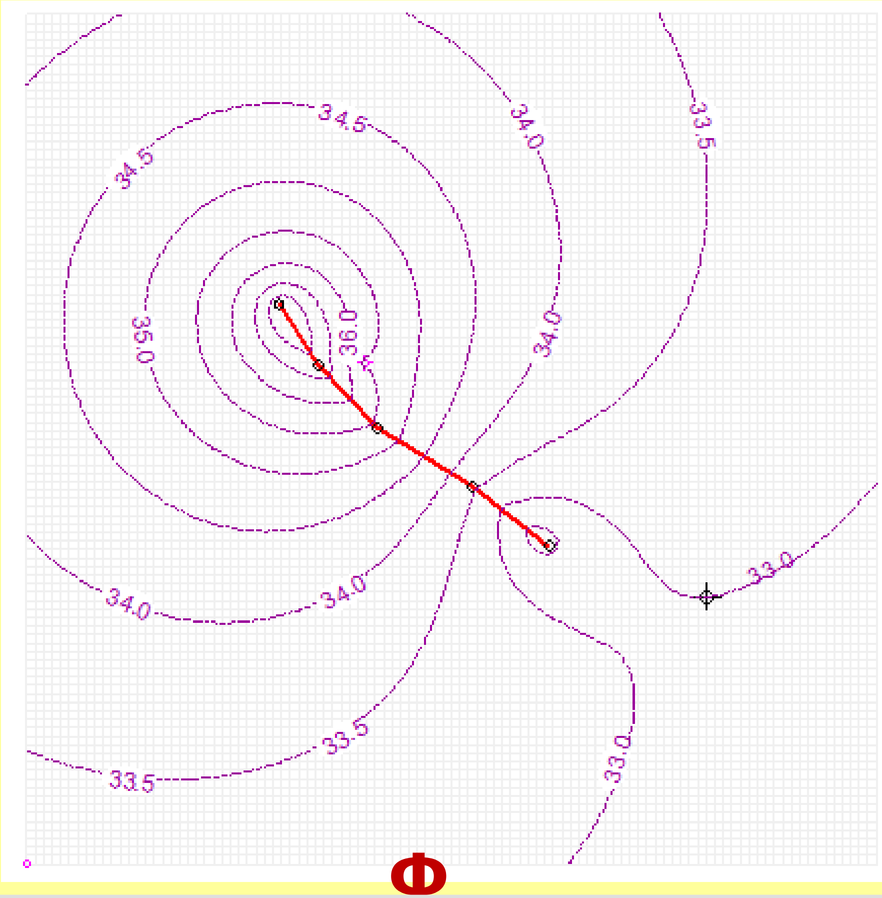
Line Element



Input : Head Specified B.C

ID	X	Y	Head (m)
1	61.5522	37.4537	32
2	52.4734	44.3822	33.58
3	41.2444	51.3107	35.42
4	34.3159	58.7171	36.82
5	29.7765	65.8845	38.0

Head Distribution and Flow vectors



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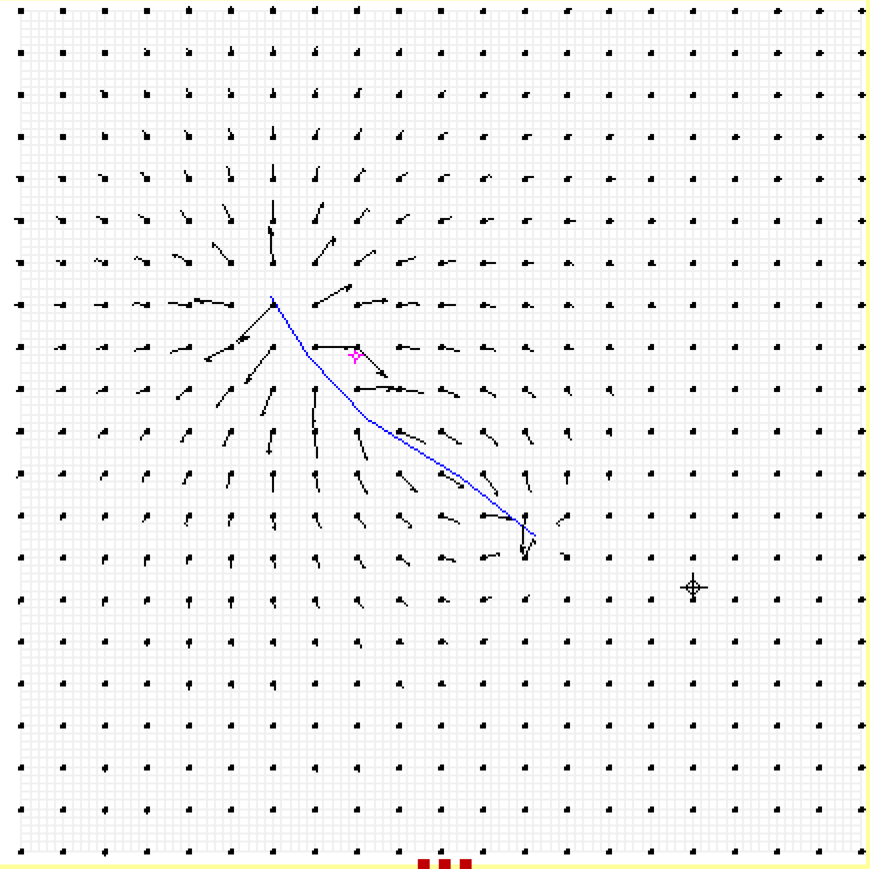

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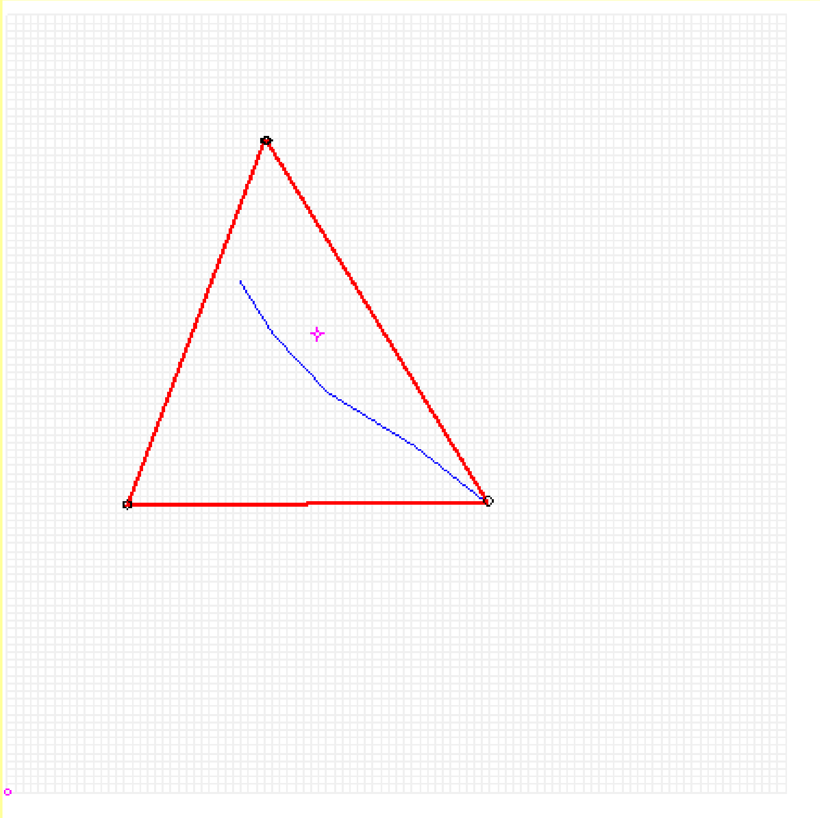
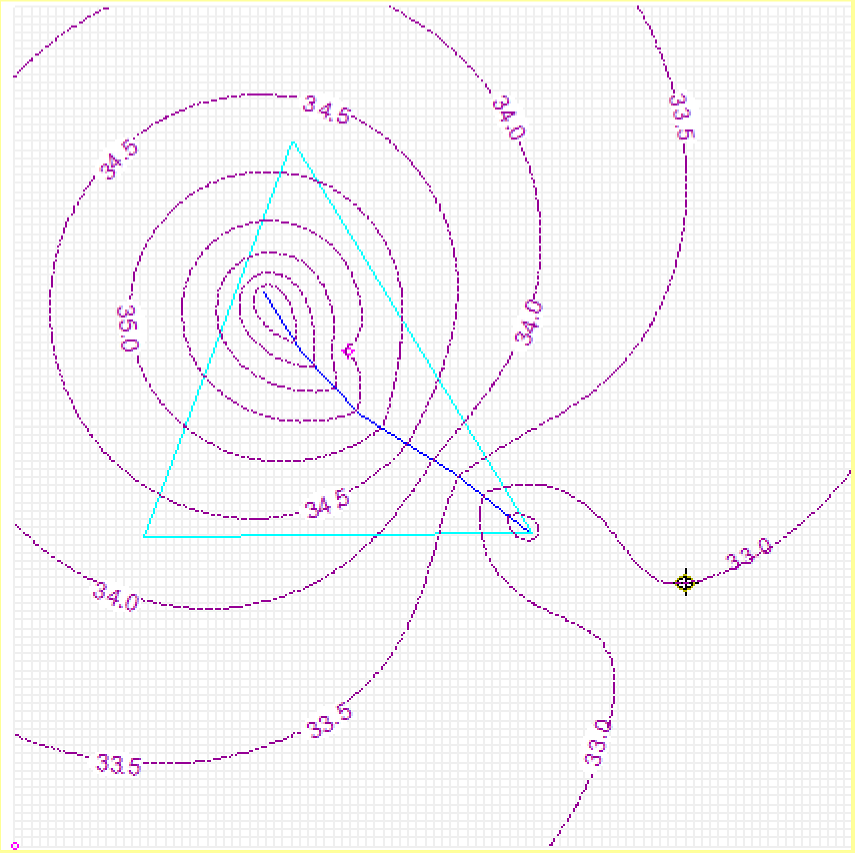
Area Element

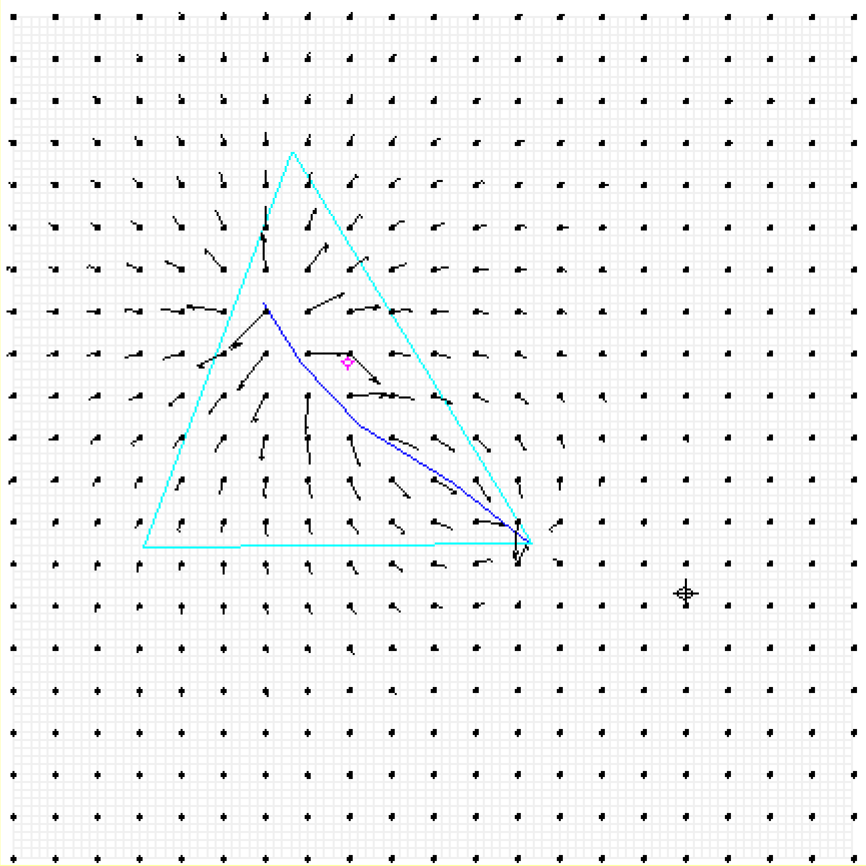


Input :

Uniform Recharge: 0.0017 m/d

Head Distribution and Flow vectors





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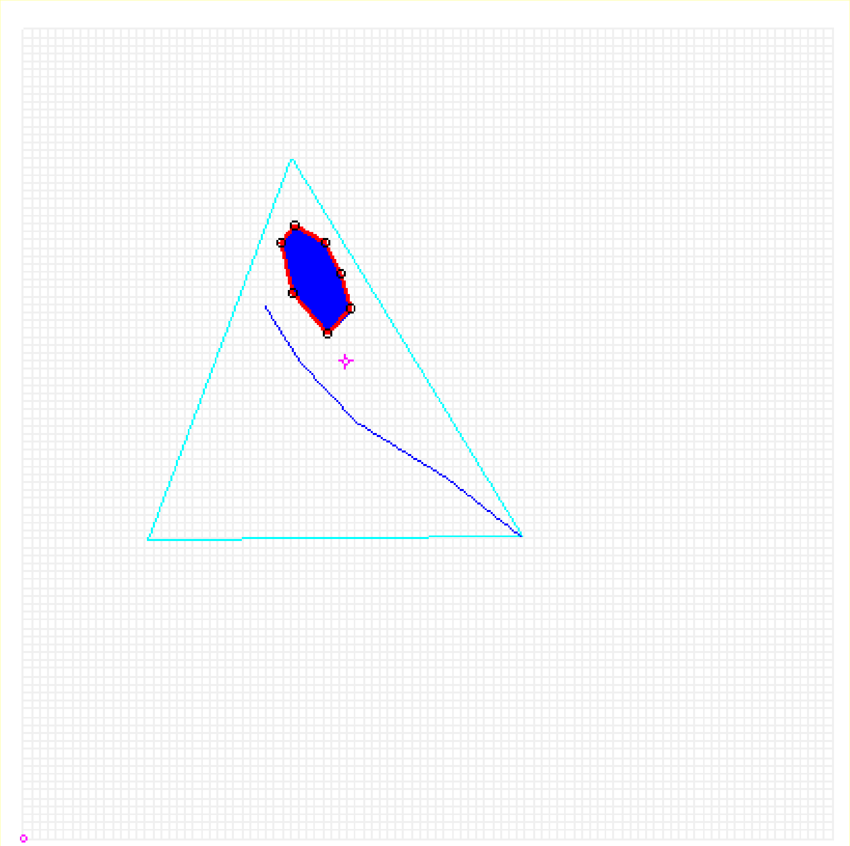
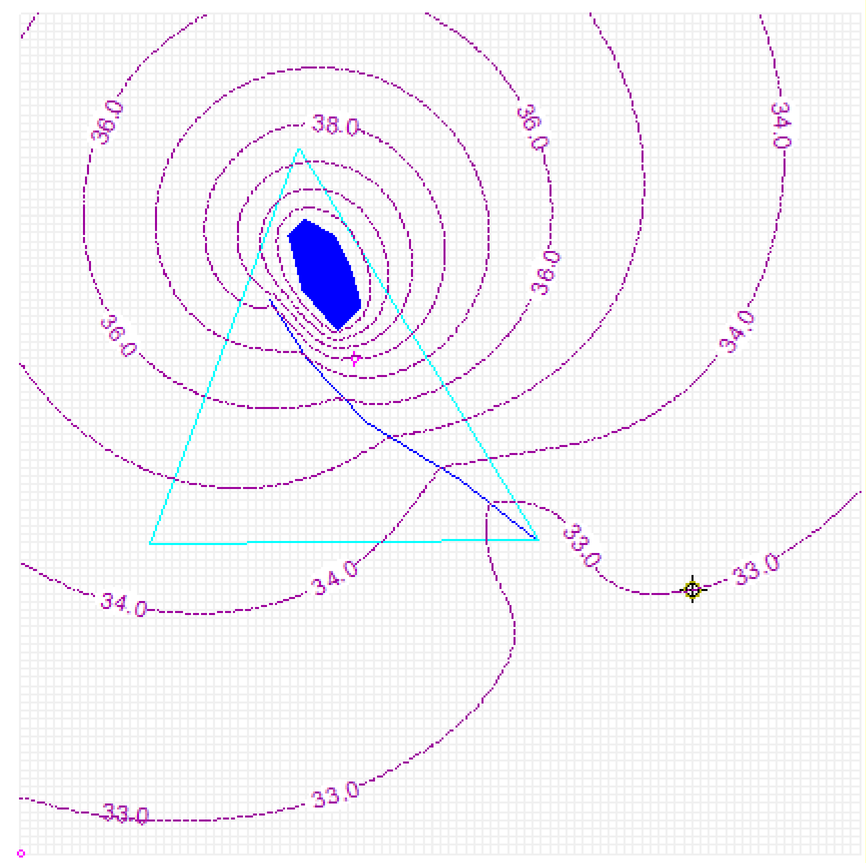
Area Element -Lake



Input :

Elevation specified lake
Lake elevation : 42 m

Head Distribution and Flow vectors



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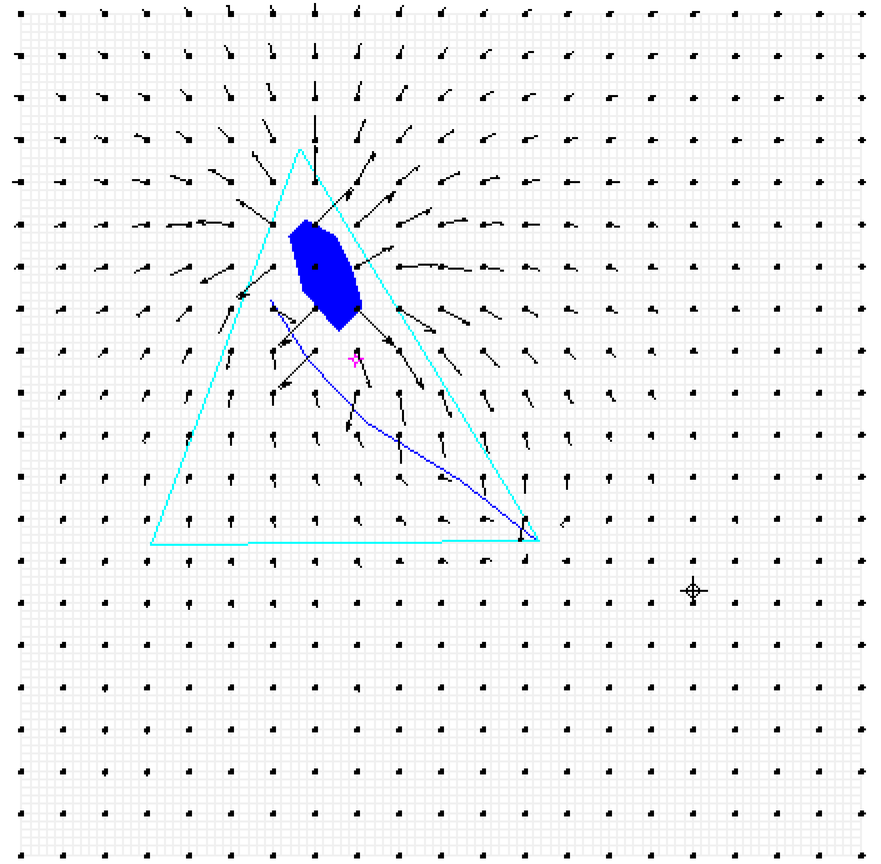

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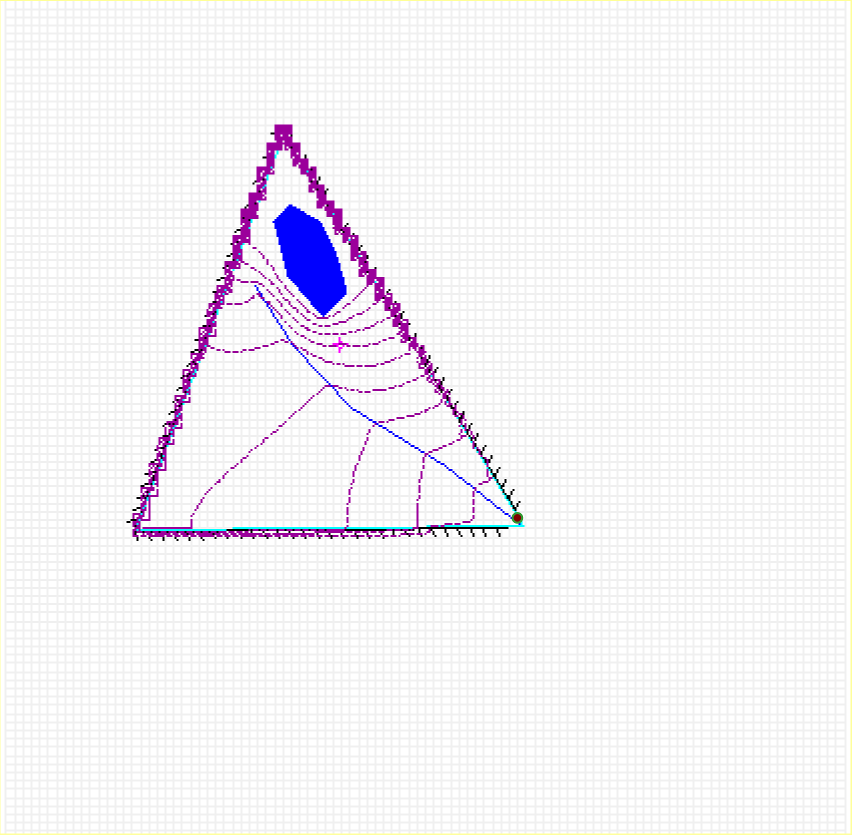
Head Distribution and Flow vectors in NO FLOW boundary



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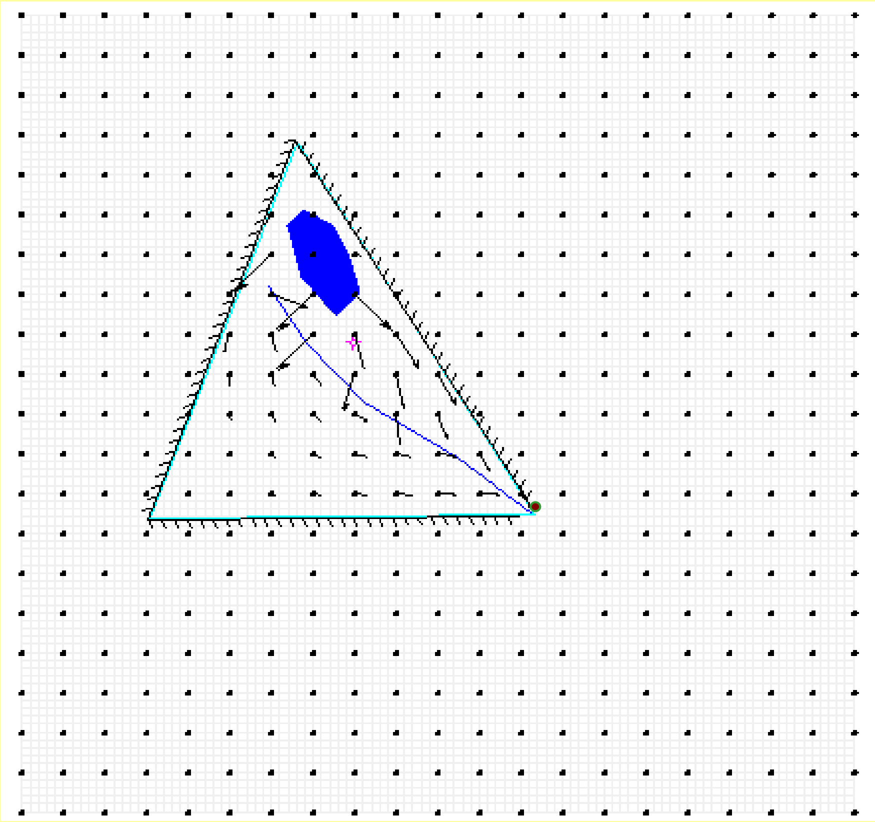

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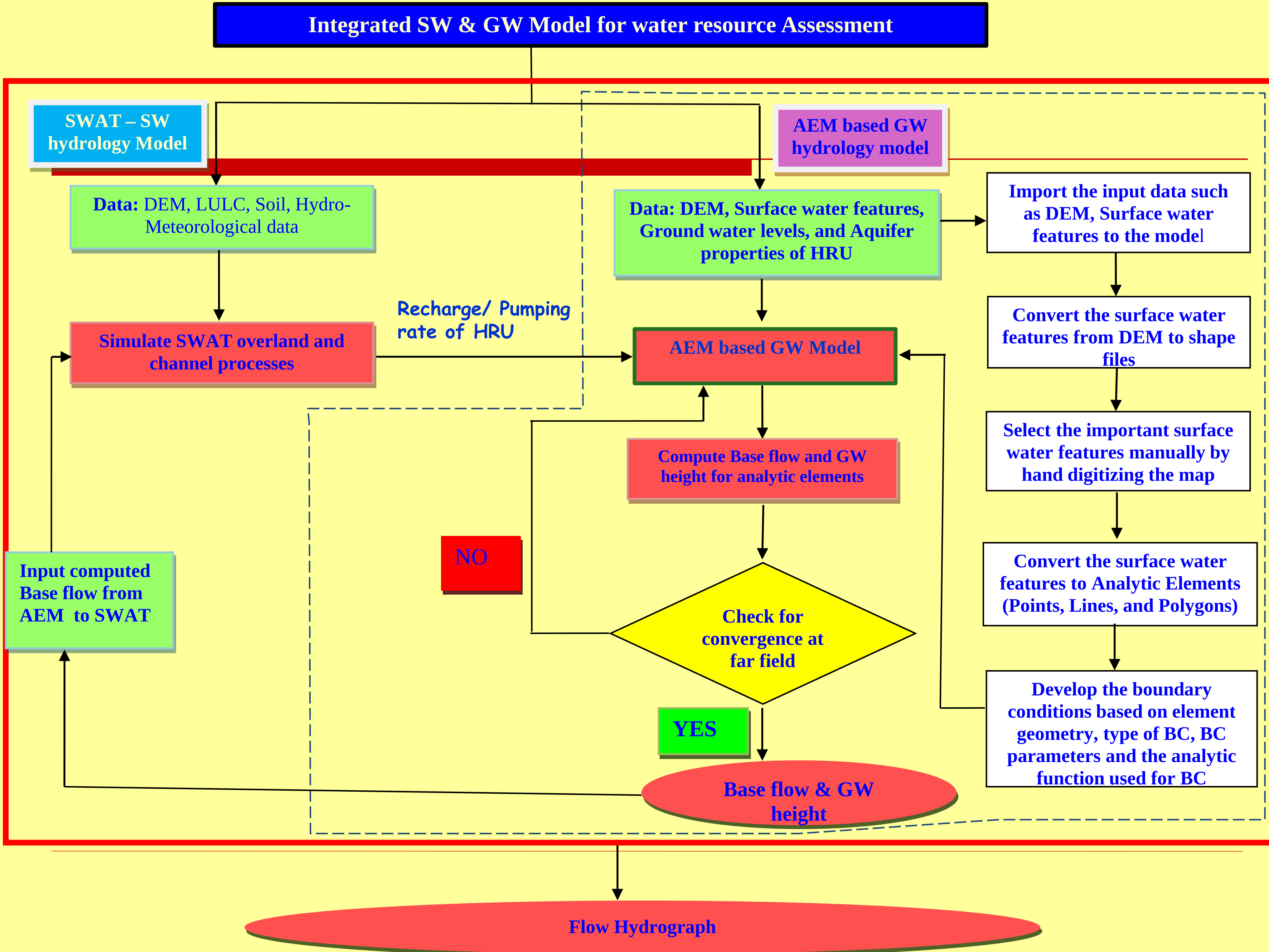
Methodology



Advantages of AEM

- Scale Independence
- Absence of grid
- Flexibility in model domain
- Computational time and less storage capacity requirement
- No need of any conversion for running the GW model
- Better Solution solvers and matrix schemes compared to FDM/FEM methods
- Continuity of flow is assured -the heads and GW velocities computed by continuous space at any point in the flow domain

Limitations of AEM

- Steady state, limited models developed for transient conditions
- Two dimensional
- Horizontal layers with constant thickness
- Homogeneity, isotropic soil
- Applicable when the equations are linear
- No vertical recharge from infiltration or leakage