Improved physically based approaches for Channel Erosion Modeling in SWAT

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Outline

- Channel Erosion
- Sediment Routing
  - In SWAT2000 and SWAT2005
- Physically based approach
  - Erosion
  - Transportation
  - Deposition
Channel Erosion

Channel erosion

- Can account for as much as 85% of total sediment yield of a watershed

Predicted loss in 3 km channel erosion = 1000 years of sheet and rill erosion at pre-conservation agriculture rates
Channel Erosion

Three major processes

- Subaerial processes
  - Climate
  - Alternate wet and dry cycles
  - Freeze/Thaw cycles
  - Cracking

- Fluvial erosion (Hydraulic Erosion)
  - Removal of particles by streamflow

- Bank Failure
  - Caused due to slope instability
**Simplified Bagnold stream power equation**

\[
conc_{sed, mx} = spcon \times v_{ch}^{sp_{exp}}
\]

\[
sed_{deg} = (conc_{sed, mx} - conc_{sed, ch})V_{ch} K_{ch} C_{ch}
\]

**Channel erosion**

- limited only by the stream power or transport capacity
- but not by limits on sediment supply from the actual erosion process
**SWAT2000 and 2005**

- **No particle size distribution of eroded sediment**
- **No bedload**
  - Hence, TSS calculated from sediment yield is often high and not directly comparable with observations.
Organic nutrient load

Are we missing to quantify a significant organic nutrient load from stream bank and attributing the nutrient loads only to overland?

Cedar Creek, Texas

- 8% of orgN and
- 15% of orgP from channel erosion
- Channel erosion – 35% of total sediment yield

Hence, accurate quantification of channel erosion is very important
Complex Processes: Simplify
Fluvial Erosion Process

For the erosion to occur

- There should be enough shear stress exerted by the flowing water on stream bank and stream bed to dislodge the sediments
- The channel should have enough stream power to carry the eroded sediments (overland + channel)
- Deposition will occur if the sediment transport capacity is low
Wash-load particle size distribution

Sediment yield from overland (MUSLE) is partitioned using the approach used in CREAMS.

\[
PSA = (SAN)(1 - CLA)^{24}
\]

\[
PSI = 0.13SIL
\]

\[
PCL = 0.20CL\]

\[
SAG = \begin{cases} 
2.0CL & \text{for } CLA < 0.25 \\
0.28(CL - 0.25) + 0.5 & \text{for } CLA \geq 0.25 \text{ and } CLA \leq 0.5 \\
0.57 & \text{for } CLA > 0.5 
\end{cases}
\]

\[
LAG = 1.0 - PSA - PSI - PCL - SAG
\]
Shear Stress

- Critical shear stress ($\tau_c$)
  - Soil parameter that governs erosion
- Excess shear stress equation:

\[
\xi_{\text{bank}} = k_{d,\text{bank}} \cdot (\tau_{e,\text{bank}} - \tau_{c,\text{bank}}) \cdot 10^{-6}
\]

\[
\xi_{\text{bed}} = k_{d,\text{bed}} \cdot (\tau_{e,\text{bed}} - \tau_{c,\text{bed}}) \cdot 10^{-6}
\]

where $\xi$ – erosion rates of the bank and bed (m/s), $k_d$ – erodibility coefficient of bank and bed (cm$^3$/N-s) and $\tau_c$ – Critical shear stress acting on bank and bed (N/m$^2$).
Shear Stress

Effective shear stress based on channel hydraulics: (Eaton and Millar, 2004)

\[
\frac{\tau_{e,\text{bank}}}{\gamma \cdot \text{depth} \cdot \text{slp}_{ch}} = \frac{SF_{\text{bank}}}{100} \left( \frac{(W + P_{\text{bed}}) \cdot \sin \theta}{4 \cdot \text{depth}} \right)
\]

\[
\frac{\tau_{e,\text{bed}}}{\gamma_w \cdot \text{depth} \cdot \text{slp}_{ch}} = \left(1 - \frac{SF_{\text{bank}}}{100}\right) \left(\frac{W}{2 \cdot P_{\text{bed}}} + 0.5\right)
\]

\[
\log SF_{\text{bank}} = -1.4026 \cdot \log \left(\frac{P_{\text{bed}}}{P_{\text{bank}}} + 1.5\right) + 2.247
\]
Critical Shear Stress and Erodibility Coefficient

Submerged Jet Test (Hanson and Cook, 1997; Hanson and Simon, 2001)

Hanson and Simon, 2001

Clark and Wynn, 2007
Empirical Equation for $\tau_c$

Critical Shear Estimates

Soil Composition

$y = 0.1 + 0.1779x + 0.0028x^2 - 2.34E-5x^3$

$n = 16$

$r^2 = 0.91$

$p < 0.001$

Range mostly between: 0 and 100 N/m²
But could go as high as 400 N/m²

Vegetation

<table>
<thead>
<tr>
<th>Type / Density</th>
<th>$\tau_c$ coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>None</td>
<td>1</td>
</tr>
<tr>
<td>Ivy / Sparse</td>
<td>1.5</td>
</tr>
<tr>
<td>Ivy / Dense</td>
<td>2.5</td>
</tr>
<tr>
<td>Privet / Sparse</td>
<td>5.4</td>
</tr>
<tr>
<td>Privet / Dense</td>
<td>19.2</td>
</tr>
</tbody>
</table>

Derived from Huang and Nanson (1998)

Data from Dunn (1959)

Julian and Torres, 2001
Empirical Equation for $K_d$

**Erodibility Coefficient, $K_d$:** (Zhu et al. 2006)

\[
K_d = 0.0034 \cdot \exp\left(\frac{0.0176}{M_e}\right)
\]

\[
M_e = \frac{((s - 1) \cdot 9.8 \cdot D_{50})^{0.5}}{(s - 1)^3 \cdot C}
\]

\[
C = 4.14 \cdot (\text{Clay}\%)^{-0.91}
\]

Where $s$ is relative density of sediment

Range mostly between 0 and 0.01 cm$^3$/N-s but could go as high as 3.75 cm$^3$/N-s for highly erodible material
Stream Power/Transport Capacity

- Four new transport equations
  - **Simplified Bagnold Equation**
    - Silt type bed material
      \[ \text{conc}_{sed,mx} = \text{spcon} \times v^{sp}_{ch} \]
  - **Kodatie model**
    - Silt to gravel size bed materials
  - **Molinas and Wu model**
    - Large sand bed rivers
  - **Yangs sand and gravel model**
    - Sand and gravel bed material
**Kodatie Model**


\[
\text{conc}_{sed,ch.mx} = \left( \frac{a \cdot v_{ch}^b \cdot y^c \cdot S^d}{Q_{in}} \right) \cdot \left( \frac{W + W_{btm}}{2} \right)
\]

Table 7.2-2. Regression coefficients for Kodatie equation

<table>
<thead>
<tr>
<th>Type of River</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silt-bed rivers (D_{50} \leq 0.05 mm)</td>
<td>281.4</td>
<td>2.622</td>
<td>0.182</td>
<td>0</td>
</tr>
<tr>
<td>Very fine to fine-bed river (0.05 mm &lt; D_{50} \leq 0.25 mm)</td>
<td>2,829.6</td>
<td>3.646</td>
<td>0.406</td>
<td>0.412</td>
</tr>
<tr>
<td>Medium to very coarse sand-bed rivers (0.25 mm &lt; D_{50} \leq 2 mm)</td>
<td>2,123.4</td>
<td>3.300</td>
<td>0.468</td>
<td>0.613</td>
</tr>
<tr>
<td>Gravel-bed rivers (D_{50} &gt; 2mm)</td>
<td>431,884.8</td>
<td>1.000</td>
<td>1.000</td>
<td>2.000</td>
</tr>
</tbody>
</table>

* a, b, c and d coefficients depend on D_{50}

*D_{50} – median bank/bed-sediment size*
Molinas and Wu Model

Molinas and Wu (2001):

\[
C_w = \frac{1430 \cdot (0.86 + \sqrt{\psi}) \cdot \psi^{1.5}}{0.016 + \psi} \cdot 10^{-6}
\]

\[
\psi = \frac{v_{ch}^3}{(S_g - 1) \cdot g \cdot \text{depth} \cdot \omega_5} \left[ \log_{10} \left( \frac{\text{depth}}{D_{50}} \right) \right]^2
\]

\[
\omega_5 = \frac{411 \cdot D_{50}^2}{3600}
\]

\[
\text{conc}_{sed,ch.mx} = \frac{C_w}{C_w + (1 - C_w) \cdot S_g} \cdot S_g
\]
**Yangs Sand and Gravel Model**

**Sand equation: (D\(_{50}\) less than 2mm):**

\[
\log C_w = 5.435 - 0.286 \log \frac{\omega_{50} D_{50}}{\nu} - 0.457 \log \frac{V^*}{\omega_{50}} \\
+ \left( 1.799 - 0.409 \log \frac{\omega_{50} D_{50}}{\nu} - 0.314 \log \frac{V^*}{\omega_{50}} \right) \log \left( \frac{V_{ch} S}{\omega_{50}} - \frac{V_{cr} S}{\omega_{50}} \right)
\]

**Gravel equation: (D\(_{50}\) between 2mm and 10mm)**

\[
\log C_w = 6.681 - 0.633 \log \frac{\omega_{50} D_{50}}{\nu} - 4.816 \log \frac{V^*}{\omega_{50}} \\
+ \left( 2.784 - 0.305 \log \frac{\omega_{50} D_{50}}{\nu} - 0.282 \log \frac{V^*}{\omega_{50}} \right) \log \left( \frac{V_{ch} S}{\omega_{50}} - \frac{V_{cr} S}{\omega_{50}} \right)
\]
Excess transport capacity

$$SedEx = V_{ch} \cdot \left( conc_{sed, ch.mx} - conc_{sed, ch.i} \right)$$

- Upper limit of channel erosion is the transport capacity or the erosion due to excess shear stress whichever is lower.
Deposition

If the sediment concentration in the channel is more than the transport capacity then deposition occurs:

Einstein equation (1965):

$$Dep_{frac} = \left(1 - \frac{1}{e^x}\right)$$

$$\omega_{50} = \frac{411 \cdot D_{50}^2}{3600}$$

$$x = \frac{1.055 \cdot L_{ch} \cdot \omega_{50}}{v_{ch} \cdot depth}$$

Flood plain deposition

If the streamflow goes overbank
Cedar Creek Watershed
Channel Erosion by various transport models

Erosion (Tons/year)

- 0 - 100
- 100 - 2000
- 2000 - 5000
- 5000 - 10000
- > 10000
Channel Deposition by various transport models

Deposition (Tons/year)
- 0 - 100
- 100 - 2000
- 2000 - 5000
- 5000 - 10000
- > 10000

Default Bagnold Kodatie Molinas Yangs
Channel Erosion

Bank/Bed Erosion (tons/year)

Sediment Transport Equations

- Default
- Bagnold
- Kodatie
- Molinas
- Yangs

CH_BNktons

CH_BEDtons
Channel Deposition

![Bar chart showing sediment deposition by model: Default, Bagnold, Kodatie, Molinas, Yangs. The chart highlights significantly higher deposition in Kodatie compared to other models.]
Total Suspended Sediments

TSSmg/L

Total Suspended Sediments (mg/L)

Sediment Transport Model

Default  Bagnold  Kodatie  Molinas  Yangs
Model Inputs

- **Default model**
  - spcon, spext, CH_cov, CH_Erod

- **Physically based models**
  - D50 – Median particle size of bank and bed material
  - Critical shear stress of bank and bed
  - Cover factor of bank and bed
  - Erodibility coefficient of bank and bed
  - Bulk density of bank and bed
  - Particle size distribution of bank and bed material
Model Output

File name: output.sed

Default
- Total sediment
- Bed erosion, deposition, TSS

Physically based models
- Total sediment
  - Sand, silt, clay, SAGG, LAGG, gravel
- Bank erosion
- Bed erosion
- Channel deposition
- Flood plain deposition
- TSS
Thank You